PRE-CALCULUS 11

Introduction Assignment

Welcome to PREC 11! This assignment will help you review some topics from a previous math course and introduce you to some of the topics that you'll be studying this year. The last part of this assignment asks you to provide your teacher with information about your previous experiences in math.

In order to earn full marks for each question, you must show all your work. Where a numerical response is required, answer to the nearest tenth.

Student Name ____________________

Student No. ____________________ **Date** ____________

Address ____________________ **Postal Code** ________

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Complete the following *Pre-Calculus 11* Assignment independently and return it to your teacher based on the instructions provided by your school. No external resources are required to complete this assignment.

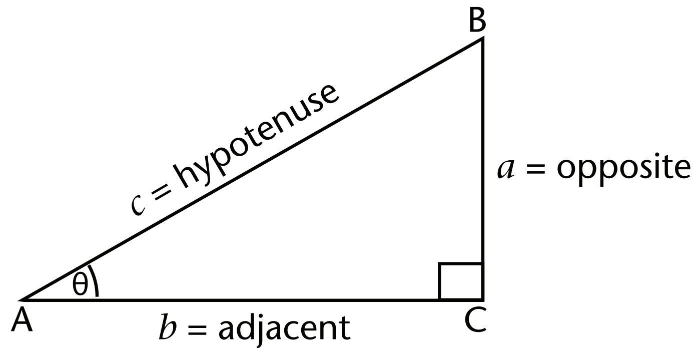
Title	Marks
Part 1: Trigonometry	/10
Part 2: Factoring Trinomials	/10
Part 3: Quadratics	/15
Part 4: Radicals	/10
Part 5: About You	/5
Total marks	/50

Contents:
24 pages

Assignment time:
3 hours

Part 1: Trigonometry (10 marks)

Triangle ABC with reference angle can be labelled with the following:



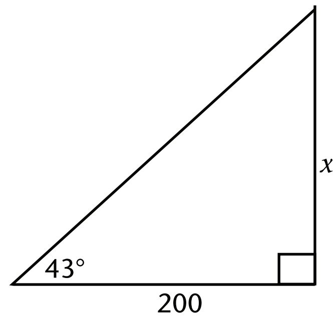
The following table summarizes the three trigonometric ratios that you learned in a previous math course.

Ratio Name	Description	Calculation	Mnemonic*
Sine	The ratio of the length of the side opposite the reference angle to the length of the hypotenuse.	$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$	S O H
Cosine	The ratio of the length of the side adjacent to the reference angle to the length of the hypotenuse.	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$	C A H
Tangent	The ratio of the length of the side opposite the reference angle to the length of the side adjacent to the reference angle.	$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$	T O A

*A way to remember

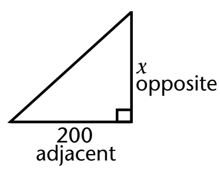
Example 1

Determine the measure of length x to the nearest tenth.



Solution

Label the triangle.



Given: one angle and the adjacent side

Need: the opposite side

Solve with tangent. SOH CAH TOA

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 43° = \frac{x}{200}$$

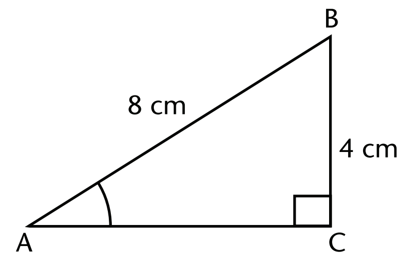
$$(200)(\tan 43°) = \left(\frac{x}{200}\right)(200)$$

$$(200)\tan(43°) = x$$

$$x = 186.5$$

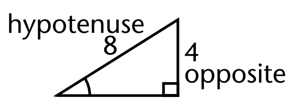
Example 2

Determine the measure of angle A to the nearest degree.



Solution

Label the triangle.



Given: the side opposite and the hypotenuse

Need: angle A

Solve with sine. SOH CAH TOA

$$
\begin{aligned}
\sin A &= \frac{\text{opposite}}{\text{hypotenuse}} \\
\sin A &= \frac{4}{8} \\
\sin A &= 0.5 \\
\angle A = \sin^{-1}(\sin A) &= \sin^{-1}(0.5) \\
\angle A &= 30^\circ
\end{aligned}
$$

To "undo" sin A and discover the measure of ∠A, find the $\sin^{-1}$ of both sides of the equation.

Solving a triangle involves finding all the remaining measurements of a triangle when you're given the measure of one length and one acute angle, or the measures of two lengths. Two things to remember:

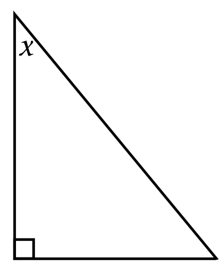
- the sum of the interior angles of a triangle is 180°, so if you're given the measure of one angle besides the right angle, you can calculate the measure of the third angle using addition and subtraction.
- you can solve for the third side of a right triangle using the Pythagorean Theorem, $a^2 + b^2 = c^2$.

MARKS

Now it's your turn.

1. Label the following triangle PQR with respect to the reference angle, x, where the right angle is at Q (there are two different ways to label it; choose one).

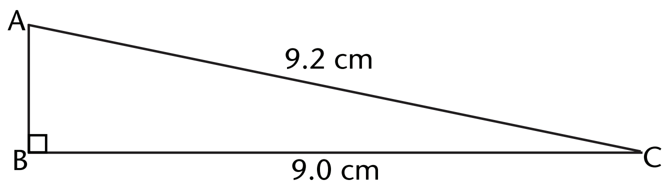
   Include the following labels: P, Q, R, p, q, r, opposite, adjacent, hypotenuse. (3 marks)



MARKS

2. Solve the following right triangles. Answer to the nearest tenth. Include units with your answers. (7 marks; 4 marks for a and 3 marks for b)

a.

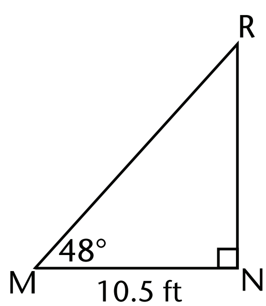


$\angle A =$ ____________

$\angle C =$ ____________

AB = ____________

b.



$\angle R =$ ____________

RN = ____________

MR = ____________

Part 2: Factoring Trinomials (10 marks)

You can factor trinomials of the form $ax^2 + bx + c$ by decomposition, using the tic-tac-toe method, or by inspection. Following are two examples showing the decomposition and tic-tac-toe methods.

Example 1

Factor $2x^2 + 7x - 15$ by decomposition.

Solution

Find two numbers whose product is equal to the product of (2)(–15), which is –30, and whose sum is equal to +7, the middle or b term.

By using a table or doing it in your head, you find that those numbers are +10 and –3.

$(+10)(-3) = -30$
$+10 + (-3) = +7$

Rewrite the trinomial as a four-term polynomial using the two numbers just found as the coefficients of the middle two terms.

$2x^2 + 10x - 3x - 15$

Group the first two and the last two terms.

$(2x^2 + 10x) + (-3x - 15)$

Factor out the common factor in each group.

$2x(x + 5) - 3(x + 5)$

Factor out the common binomial.

$(2x - 3)(x + 5)$

Example 2

Factor $3x^2 - 8x + 4$ using the tic-tac-toe method.

Solution

Find two numbers whose product is (3)(4), which is 12, and whose sum is –8. These numbers are –6 and –2.

Construct a 3 × 3 array as shown. Place the x^2-term of the trinomial in the centre square and the constant in the last square in the bottom row.

	$3x^2$	
		$+4$

Place an x-term in the last square in the middle row, and an x-term in the second square of the last row. These x-terms should have coefficients equal to the numbers determined earlier.

	$3x^2$	$-6x$
	$-2x$	$+4$

Now find the Greatest Common Factor (GCF) of each row and column and write them in the row and column headings. For now, make them all positive.

GCF of $3x^2$ and $-2x$ is x. GCF of $-6x$ and 4 is 2.

		x	2
GCF of $3x^2$ and $-6x$ is $3x$.	$3x$	$3x^2$	$-6x$
GCF of $-2x$ and 3 is 2.	2	$-2x$	$+4$

Now we'll check the sign of each GCF. $3x^2$ is positive, so x and $3x$ can stay positive. However, the product of $3x$ and the other factor (currently +2) must be $-6x$, so we'll make the 2 negative. In the same way, the product of x and the other factor must be $-2x$, so we'll make the other 2 negative as well. Our final chart looks like this:

	x	-2
$3x$	$3x^2$	$-6x$
-2	$-2x$	$+4$

Now you can read the two binomial factors from the top row and the left-most column of the grid.

$$(x-2)(3x-2) = 3x^2 - 8x + 4$$

When factoring trinomials, you may also see a special case called the difference of squares. It's like a trinomial where the middle term has a coefficient of zero. Here are some examples.

$x^2 - 49$ $\quad\quad$ $4x^2 - 25$ $\quad\quad$ $16m^2 - 81$

To factor a difference of squares, take the square root of each term. Then write one factor as the sum of the square roots and the other factor as the difference of the square roots.

Example 3

Factor $9x^2 - 4y^2$ completely and check by multiplying.

Solution

Step 1: First, check the terms for common factors. Since there are none, continue by determining the square roots of $9x^2$ and $4y^2$.

$$\sqrt{9x^2} = 3x$$

$$\sqrt{4y^2} = 2y$$

Step 2: Write one factor as the sum of the square roots. Write the other factors as the difference of the square roots.

$$9x^2 - 4y^2 = (3x+2y)(3x-2y)$$

MARKS

Step 3: Check by multiplying.

$$\begin{aligned}&(3x+2y)(3x-2y)\\&=3x(3x-2y)+2y(3x-2y)\\&=9x^2-6xy+6xy-4y^2\\&=9x^2-4y^2\end{aligned}$$

The original difference of squares is recovered, so the factors are correct.

As the example above suggests, before you start factoring by decomposition or using the tic-tac-toe method, check for a common factor in each term.

Now it's your turn.

1. Factor the following. (10 marks; 1 mark each)

   $x^2 + x - 6$

   $2x^2 + 11x + 12$

   $5x^2 - 7x + 2$

   $2x^2 + 7x + 6$

   $6x^2 + 11x - 10$

   $x^2 + 6x + 9$

MARKS

$2x^2 - 13x + 6$

$x^2 - 6x + 8$

$4x^2 + 12x + 9$

$81x^2 - 144y^2$

MARKS

Part 3: Quadratics (15 marks)

In PREC 11 you'll be studying quadratic functions and equations. This introduction will help you review your graphing and analytical skills.

The vertex form of a quadratic function is

$$f(x) = a(x - p)^2 + q$$

where a, p, and q are constants and $a \neq 0$

Here are two examples of quadratic equations.

$y = -2(x - 1)^2 + 4$ $\qquad$ $y = \frac{1}{2}(x + 2)^2 - 1$

1. Create a table of values for each equation by substituting various values of x and calculating the corresponding y-values. Use the x-values given in each table. (4 marks)

   Note: Show your work for the first calculation in each table, but after that you can do further calculations on scrap paper, on your calculator, or in your head, if you want.

   a. $y = -2(x - 1)^2 + 4$

x	y
1	
0	
2	
–1	
3	–4
–2	
4	

MARKS

b. $y = ½(x + 2)^2 - 1$

x	y
–2	
–1	
–3	
0	
–4	
1	3½
–5	
2	
–6	

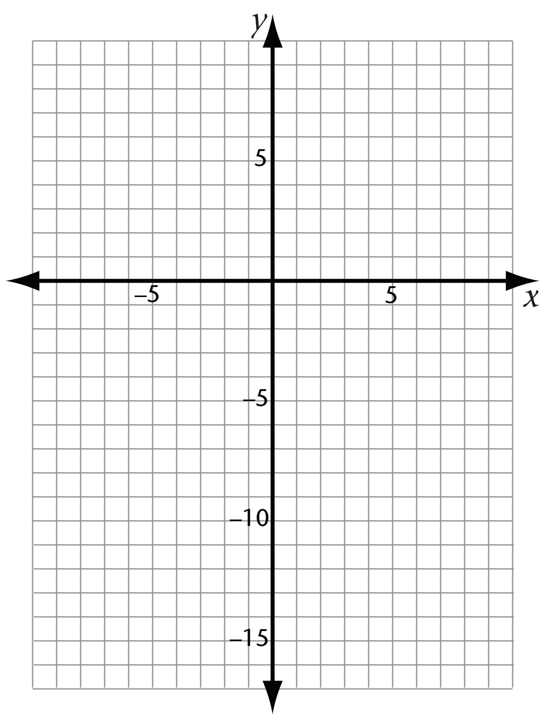
MARKS

2. Using the tables of values from above, sketch both of the graphs. (5 marks)

   Note the following:

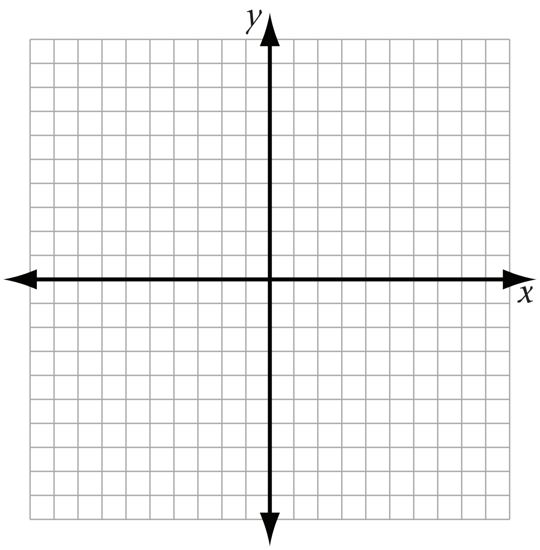
   - The graphs are continuous, so join the points with lines.
   - The graphs continue past the points we're graphing, so put arrows on the ends of the lines.

   a. $y = -2(x - 1)^2 + 4$



MARKS

b. $y = \frac{1}{2}(x + 2)^2 - 1$



3. Analyze the graphs by answering the following. Write at least two points for each question. (4 marks)

   a. What is similar in both graphs?

   ____

   ____

   ____

   b. What is different about the graphs?

   ____

   ____

   ____

MARKS

4. Each graph above has a vertex, the lowest or highest point on the graph (depending on whether it “opens upward” or “opens downward”).

   What is the vertex for each graph? (2 marks)

   $y = -2(x - 1)^2 + 4$ ____________________

   $y = \frac{1}{2}(x + 2)^2 - 1$ ____________________

Part 4: Radicals (10 marks)

Converting Entire Radicals to Mixed Radicals (aka simplifying radicals)
Answer the following questions about your last math course.

One of the properties of radicals is the following:

$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$, where a and b are positive numbers.

For instance, the expression $\sqrt{4 \times 3}$ can be also written as $\sqrt{4} \times \sqrt{3}$.

You can use this property to simplify radicals.

If a radical of index 2 has a perfect square among its factors:

- Rewrite the radicand as a product of two factors, one of which is the perfect square (e.g., 4, 9, 16, 25, 36, 49, 64, 81, 100, ...).
- Take the square root of the perfect square: it becomes the coefficient of the simplified radical.

Example 1

Simplify the radical $\sqrt{48}$.

$$\begin{aligned} &\sqrt{48} \\ &= \sqrt{16 \times 3} \\ &= \sqrt{16} \times \sqrt{3} \\ &= 4\sqrt{3} \end{aligned}$$

Simplify $\sqrt[3]{54}$.

$$\begin{aligned} \sqrt[3]{54} &= \sqrt[3]{3 \times 3 \times 3 \times 2} \\ &= \sqrt[3]{3 \times 3 \times 3} \times \sqrt[3]{2} \\ &= 3\sqrt[3]{2} \end{aligned}$$

Converting Mixed Radicals to Entire Radicals
Entire radicals do not have coefficients. To convert a mixed radical to an entire radical, we need to change the coefficient to the equivalent number under a radical sign.

Example 1

Write $2\sqrt{5}$ as an entire radical.

Solution
We need to write the 2 as a number under a radical sign.

Method 1: Perfect Square Factors

The number 2 can be written as $\sqrt{2^2}$ or $\sqrt{4}$.

Then $2\sqrt{5}$
$= \sqrt{4} \times \sqrt{5}$
$= \sqrt{20}$

Method 2: Prime Factorization

Remember that the coefficient of 2 came from a pair of 2s under the radical symbol.

Write 2 as $\sqrt{2 \times 2}$.

Then $2\sqrt{5}$
$= \sqrt{2 \times 2 \times 5}$
$= \sqrt{20}$

Example 2

Write $4\sqrt[3]{5}$ as an entire radical.

Solution
Using the method of prime factorization, write 4 as

$\sqrt[3]{4 \times 4 \times 4}$ or $\sqrt[3]{64}$

Then $4\sqrt[3]{5}$
$= \sqrt[3]{64} \times \sqrt[3]{5}$
$= \sqrt[3]{64 \times 5}$
$= \sqrt[3]{320}$

MARKS

Now it's your turn.

1. Convert the following entire radicals to mixed radicals (simplify). (5 marks; ½ mark each)

a. $\sqrt{20}$

b. $\sqrt{45}$

c. $\sqrt{72}$

d. $\sqrt{175}$

e. $\sqrt[3]{16}$

f. $\sqrt[3]{375}$

g. $\sqrt{150}$

h. $\sqrt[4]{48}$

i. $\sqrt{112}$

j. $\sqrt[3]{189}$

MARKS

2. Convert the following mixed radicals to entire radicals. (5 marks; ½ mark each)

a. $2\sqrt{3}$

b. $3\sqrt{5}$

c. $3\sqrt{2}$

d. $2\sqrt[3]{5}$

e. $6\sqrt{3}$

f. $3\sqrt[3]{6}$

g. $2\sqrt[4]{2}$

h. $2\sqrt{15}$

i. $2\sqrt[3]{7}$

j. $2\sqrt{5}$

Part 5: About You (5 marks)

Answer the following questions about your last math course.

What was the name of your last math course? ______________________

____

Explain the format of the course (e.g., in a classroom, online, print-based distance learning, etc.)

____

____

When did you finish it? ______________________

How long did it take you to complete the course? ______________________

____

Did you do well in it? ______________________

Explain: ______________________

____

What did you like best about your last math course? ______________________

____

____

Are there certain topics that you find challenging or difficult?

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____

____

What do expect to achieve in PREC 11?

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____

Please add anything else about yourself or your previous experiences in math that may help your teacher guide you through this course.

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/10	Part 1: Trigonometry
/10	Part 2: Factoring Trinomials
/15	Part 3: Quadratics
/10	Part 4: Radicals
/5	Part 5: About You
50	**Total**