Foundations of Mathematics and Pre-calculus 10

Module 1 Blackline Masters

This blackline master package, which includes all section assignments, as well as selected worksheets, activities, and other materials for teachers to make their own overhead transparencies or photocopies, is designed to accompany Open School BC's Foundations of Mathematics and Pre-calculus (FMP) 10 course. BC teachers, instructional designers, graphic artists, and multimedia experts developed the course and blackline masters.

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Foundations of Mathematics and Pre-calculus 10

Module 1, Section 1—Lesson A: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
How can you use referents to estimate measurements?			
Why are there two systems of measurement?			

	Definition	Metric	Imperial	Referent and Explanation
Bolt				
Furlong				
League				
Milestone				
Chain				

Section 1—Lesson A: Going Beyond

Referents Table

Units	Description and Possible Referent	Example
	The millimetre is $\frac{1}{1000}$ or 0.001 of a metre.	
Millimetre	It's roughly the thickness of a	
	1 mm is the same as about 10 sheets of paper.	9 9 19 19 19 19 19 19 19 19 19 19 19 19
	The centimetre is $\frac{1}{100}$ or 0.01 of a metre.	allunitum s made 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
Centimetre	It's approximately the thickness of 10 dimes.	In the second
	1 cm is about the width of the fingernail on your smallest finger.	
Metre	One metre is the same as the height of a small child 1 m is also about the length of a long walking stride.	
	One kilometre is equal to 1000 m.	
Kilometre	1 km is about the length of 7 football fields.	
	It's also the distance you can walk in about 10 minutes.	
Inch	One inch is an imperial unit. 1 in can be approximated by the length of your thumb from the last joint to the end. It can also be shown as 1".	
	One foot is equal to 12 inches	Photo by Smart Foto © 2010
	There's no better referent for a foot than your own foot!	
Foot	1 foot can also be approximated by the length from your wrist to your elbow.	
	A convenient referent for 1 ft is the length of a 30 cm ruler. It can also be shown as 1'.	
	One yard is equal to 3 ft or 36 in.	7
Yard	1 yard is about the width of an average doorway.	· · · · · · · · · · · · · · · · · · ·
	1 yd is also about the same as the length of a normal walking stride.	Photo by Zdenek Krchak © 2010
	One mile is equal to 1760 yards or 5280 feet.	
Mile	One mile is the distance of 15 to 20 city blocks.	
	1 mile is about the distance the average person can jog in 10 minutes.	Jack here

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Module 1, Section 1—Lesson C: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
How are the strategies for converting units in the SI different from those used in the imperial system?			
When can you use proportions to solve problems?			

_____ Student No. _____

Section 1 Assignment: Math Lab: Body Referents

Procedure

1. Complete the table below by measuring each body part described. Record your measurements in both centimetres (SI units) and inches (imperial units). (10 marks)

Body Part	Explanation	SI Measure (cm)	Imperial Measure (in)
Digit	width of fingernail		
Thumb nail	width of thumbnail		
Palm	distance across palm		
Span	hand width, from outspread thumb to little finger		
Foot	foot length, from heel to toe (good for measuring short distances)		
Cubit	elbow to fingertip		
Yard	nose to fingertip, with arm straight out to side, head facing front		
Fathom	fingertip to fingertip, arms outstretched		
Step	one step		
Расе	two steps (good for measuring long distances)		

Your	Name	
------	------	--

Student No. ___

- 3. Estimate the length of the items listed below using a referent. Record the name of the object, the referent you used (e.g., thumbnail, foot, cubit) and your estimates, both in SI units and imperial units, in the Estimates columns of the table. (10 marks)
 - your desk or your kitchen table
 - height of a wall
 - length of an object that you can hold in your hand, such as your calculator
 - distance from one corner of the room to the opposite corner
 - perimeter of the classroom or other room

Object	Referent Used	Estimate (SI)	Actual (SI)	Estimate (Imperial)	Actual (Imperial)

5. Now measure each of the lengths, heights, and distances using a tape measure or ruler. Fill in the Actual columns in the table so that you can compare the results to your estimated values.

Your Name _____ Student No. _____

Analysis

Answer the following the questions.

- 6. How close were your estimated values to the measured values? (1 mark)
- 7. What strategy could you use to make your estimation better? (1 mark)

8. What benefits are there when you measure using referents? (1 mark)

9. What drawbacks are there when you measure using referents? (1 mark)

Section Assignment 1.1

Math Lab: Body Referents

Procedure

1. Complete the table below by measuring each body part described. Record your measurements in both centimetres (SI units) and inches (imperial units). (10 marks)

Body Part	Explanation	SI Measure (cm)	Imperial Measure (in)
Digit	width of fingernail		
Thumb nail	width of thumbnail		
Palm	distance across palm		
Span	hand width, from outspread thumb to little finger		
Foot	foot length, from heel to toe (good for measuring short distances)		
Cubit	elbow to fingertip		
Yard	nose to fingertip, with arm straight out to side, head facing front		
Fathom	fingertip to fingertip, arms outstretched		
Step	one step		
Pace	two steps (good for measuring long distances)		

- 2. Estimate the length of the items listed below using a referent. Record the name of the object, the referent you used (e.g., thumbnail, foot, cubit) and your estimates, both in SI units and imperial units, in the Estimates columns of the table. (10 marks)
 - your desk or your kitchen table
 - height of a wall
 - length of an object that you can hold in your hand, such as your calculator
 - distance from one corner of the room to the opposite corner
 - perimeter of the classroom or other room

Object	Referent Used	Estimate (SI)	Actual (SI)	Estimate (Imperial)	Actual (Imperial)

3. Now measure each of the lengths, heights, and distances using a tape measure or ruler. Fill in the Actual columns in the table so that you can compare the results to your estimated values.

Analysis

Answer the following the questions.

4. How close were your estimated values to the measured values? (1 mark)

5. What strategy could you use to make your estimation better? (1 mark)

6. What benefits are there when you measure using referents? (1 mark)

7. What drawbacks are there when you measure using referents? (1 mark)

Section Assignment 1.1 Part 1 Systems of Measurement

- 1. Often you need to estimate the size of an object. The object can be very large or very small. Explain how you would estimate the following objects and provide the estimation (10 marks; 2 marks each)
 - a. The length of your pen

b. The distance across a baseball diamond

c. The length of your yard

d. The thickness of a quarter

e. The distance from your home to the next town

2. For the back country chalet shown, explain how you could use referents to estimate the height the building in both imperial and SI units. (2 marks)



 Name two irregular shapes and describe how you would measure them. (4 marks) Section Assignment 1.1 Part 2 Using Measurement Instruments

1. Draw a circle by tracing around the bottom of a mug or other circular object. (1 mark)

a. Estimate the length of the circumference. _____ (1 mark)

- b. Estimate the length of the diameter. _____ (1 mark)
- 2. Next, use a tape measure or a ruler to measure the diameter of the circle. Write the measurement on the diagram above. (1 mark)
- 3. Calculate the circumference of the circle using the formula. (2 marks)

4. Using a string and ruler or another method of your own, measure the circumference of the circle that you drew in question 1. Write the measurement here:

_____ (2 marks)

a. What method did you use? (1 mark)

b. Compare your answer with the one you got in question 3. (2 marks)

c. Which method do you think is more accurate? Why? (2 marks)

5. Measure the diameter of a dime using the online vernier caliper. You can carefully hold the dime up against your computer screen. According to the online caliper, what is the diameter of the dime? (2 marks)

- 6. Discuss the appropriateness of using each instrument to measure the given length or distance. If the instrument won't work, describe how best to measure the object. (6 marks; 2 marks each)
 - 1. the diameter of a dime with trundle wheel

Appropriate instrument?

Alternative way to measure?

2. the circumference of a soup can with a vernier caliper

Appropriate instrument?

Alternative way to measure?

3. the perimeter of a room with a piece of string and a tape measure

Appropriate instrument?

Alternative way to measure?

Section Assignment 1.1 Part 3

Conversions

- 1. Convert the following imperial units. (6 marks-1 mark each)
 - a. 4 feet to inches
 - b. 60 inches to feet
 - c. 18 feet to yards
 - d. 56 inches to feet and inches
 - e. 43 feet to yards and feet

6723 inches to yards, feet and inches

- 2. Convert the following: (3 marks)
 - a. 215 mm to cm
 - b. 7960 m into km
 - c. 1.33 km into cm
- 3. A large rectangular photograph has dimensions 102 cm by 55 cm. Determine the perimeter of the photograph in metres to the nearest tenth of a metre. (2 marks)

4. A large rectangular photograph has dimensions 102 cm by 55 cm. Determine the perimeter of the photograph in metres to the nearest tenth of a metre. (2 marks)

 The distance from Victoria, BC to Halifax, NS is about 6185 km. The distance from Albany, NY to San Francisco, CA is about 2980 miles. Which distance is greater? Support your answer. (2 marks)

6. You have found that you weigh 60 kilograms. If 1 kilogram = 2.2 pounds, how much do you weigh in pounds? (1 mark)

Section Assignment 1.1 Part 4 Glossary

Write a short definition from your personal glossary for each term below. (8 marks; 1 mark each)

- imperial measurement
- referent
- SI measurement
- circumference
- trundle wheel
- vernier caliper
- unit analysis
- conversion factor

Section Assignment 1.1 Part 5 Multiple Choice

20 marks: 2 marks each

No calculator may be used for this part of the section assignment.

1. Which of the following calculations converts 3754 feet to kilometres?

a.
$$3754 \text{ ft} \times \frac{1 \text{ km}}{1000 \text{ m}}$$

b. $3754 \text{ ft} \times \frac{0.3048 \text{ m}}{1 \text{ ft}} \times \frac{1 \text{ km}}{1000 \text{ m}}$
c. $3754 \text{ ft} \times \frac{5280 \text{ ft}}{1 \text{ mile}} \times \frac{1 \text{ mile}}{1.609 \text{ km}}$

- a. A only
- b. B only
- c. C only
- d. B and C
- 2. What is the diameter of the pipe shown below?



3. Sarah and Jeremy worked out two different solutions to a question when asked to change a measurement from 2 feet 4 inches to centimetres.

Sarah's solution	$\begin{split} &4 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} \approx 0.33 \text{ ft} \\ &2.33 \text{ ft} \times \frac{0.3048 \text{ m}}{1 \text{ ft}} \times \frac{100 \text{ cm}}{1 \text{ m}} \approx 71.02 \text{ cm} \end{split}$
Jeremy's solution	2 ft 4 in 2 ft × $\frac{12 \text{ in}}{1 \text{ ft}}$ = 24 in 24 in × $\frac{2.54 \text{ cm}}{1 \text{ in}}$ ≈ 60.96 cm

- a. Neither is correct.
- b. Only Sarah is correct.
- c. Only Jeremy is correct.
- d. Both are correct.
- 4. As an estimation strategy, which of the following would best be used to approximate 1 foot?
 - a. The width of your hand
 - b. The width of your fingernail
 - c. The length of your arm from elbow to fingers
 - d. One long stride

5. Using the diagram, find the length of the pencil.



You may use your calculator for the last five questions.

6. Jennifer thinks that she's made a mistake in converting a measurement in metres to yards, feet, and inches. Analyze her work and find where she might have made the mistake.

Line Number	Solution	a. b.	Lines I and III Lines II and IV
I	$4.5 \ m \!\times\! \frac{1 \ yd}{1.609 \ m} \!\approx\! 2.797 \ yd$	c. d.	Lines II and III Her answer is correct—she made no mistakes
П	$0.797 \ yd \times \frac{3 \ ft}{1 \ yd} \approx 2.391 \ ft$		
111	$0.391 \ yd \times \frac{12 \ in}{1 \ ft} \approx 4.5 \ in$		
IV	$4.5\ m\approx 2\ yd, 2\ ft,\ 5\ in$		

7. Which measurement is the largest?

14 ft 3 in	430 cm	0.429 m	160 in

- a. 14 ft 3 in
- b. 430 cm
- c. 0.429 m
- d. 160 in
- 8. A container that Jacey has to order from the United States only comes in imperial measurements. The container must be big enough to hold a flat circular object with a diameter of 15 cm. What are the length and width of the smallest container that she can order?
 - a. $10^{\prime\prime} \times 12^{\prime\prime}$
 - b. $8'' \times 10''$
 - c. $5'' \times 7''$
 - $d. \quad 4'' \times 6''$
- 9. Convert 10 yd, 2 ft, 2 in, to metres.
 - a. 11.7 m
 - b. 9.1 m
 - c. 9.7 m
 - d. 9.8 m

10. Three students converted 5849 m to an approximate imperial measurement.

Jason	$5849~m\times\frac{1~yd}{0.9144~m}\approx 6397~yd$
Ricki	$5849 \text{ m} \times \frac{1 \text{ yd}}{0.9144 \text{ m}} \times \frac{1 \text{ mile}}{1760 \text{ yd}} \approx 3.63 \text{ miles}$
Sam	$5849\ m \times 0.9144\ m \times \frac{1\ mile}{1760\ yd} \approx 3.04\ miles$

Which solution is correct?

- a. Only Jason's.
- b. Only Ricki's.
- c. Only Sam's.
- d. Both Jason's and Ricki's

Title	Marks
Math Lab: Body Referents	/24
Part 1: Systems of Measurement and Personal Referents	/16
Part 2: Using Measurement Instruments	/21
Part 3: Basic Measurement Systems and Conversions	/13
Part 4: Glossary	/8
Part 5: Multiple Choice	/20
Total Marks	/102

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Module 1, Section 2—Lesson A: Essential Questions

Essential	Before the Lesson:	After the Lesson:	Examples
Questions	What I Know	What I Learned	
How do you find the surface area of a 3-D object such as a cone, cylinder, prism, pyramid, or sphere?			

My Notes

Discover

Math Lab: The Surface Area of an Orange



Photo by Max Krasnov © 2010

Purpose

In this investigation you'll determine the relationship between the surface area of a sphere and its radius.

Materials

- orange (the rounder the better)
- string
- ruler
- 1 cm × 1 cm Grid Paper (from the Appendix)

Procedure

Step 1: Measure the circumference of the orange in centimetres by wrapping a string around the circumference. Then measure the string with a ruler. Take three measurements in total, each time wrapping the string around the orange in a different direction. Record your results in the chart below.

Measurement	Circumference
1	
2	
3	
Average	

- **Step 2:** Average the three measurements to obtain the average circumference of the orange. Record the average in the table.
- **Step 3:** Peel the orange into four to six pieces of peel.
- **Step 4:** Flatten the peels onto the $1 \text{ cm} \times 1 \text{ cm}$ Grid Paper.
- **Step 5:** Trace the peels onto the grid paper; then discard the peels.
- Step 6: Count the number of 1 cm × 1 cm squares covered by the peels that you traced. Where a full square is not covered, count two semi-covered squares as one.
- **Step 7:** Record the total area of the peels in the Surface Area, A, column of the table.

	Diameter, d	d ²	Surface Area, A	$\frac{A}{d^2}$
Your orange				
Sample 1	7.4166		172 cm ²	
Sample 2	6.9392		153 cm ²	
Sample 3	6.9073		149 cm ²	
Average				

Analysis

- 1. Use the average measurement and the circumference formula, $C = \pi d$, to calculate the diameter of your orange. Record the result in the table.
- 2. Calculate d^2 for your orange and record this number in the chart. Then divide the area of the peel (*A*) by the square of the diameter of the orange (d^2). Record the results of the calculation in the last column of the table.
- 3. Finish the calculations for d^2 and $\frac{A}{d^2}$ for the three samples oranges.
- 4. Calculate the averages of the results from all of the oranges and fill in the last row of the table. What special mathematical number does the result approximate?

My Notes

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Module 1, Section 2—Lesson B: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
How do you find the volume of 3-D objects such as prisms, pyramids, cones and cylinders?			
How are the formulas for the volumes of prisms, pyramids, cones and cylinders related to each other?			

My Notes

Discover

Math Lab Part 1: Volume of a Cone

The volume of a cone is related to the volume of a cylinder. Do the short investigation below to see how the volume of a cone and a cylinder with the same height and radius are related.

Purpose

To compare the volume of a cone to the volume of a cylinder.

Materials

- dry, empty soup can or mug with straight sides
- fine-grained cereal or rice
- loose-leaf paper
- tape
- scissors
- felt pen or pencil

Procedure

- Step 1: Roll the loose-leaf paper into the shape of a cone so that its tip touches the middle of the bottom of the can or mug. Ensure that the tip is pointed and does not have a large gap.
- **Step 2:** Allow the cone to spread wider until the side is touching the opening of the can the way that the illustration below demonstrates.



- **Step 3:** Holding the loose-leaf paper together, remove the cone from the can. Then tape the paper together in the shape of the cone.
- **Step 4:** Place the cone back in the can, and use the felt pen to draw a line all the way around the cone where the lip of the can meets the cone.
- Step 5: Use the scissors to cut along this line. You should now have a cone that has the same radius and height as the soup can.



- **Step 6:** Fill the cone to the top edge with cereal. Make sure the top of the cereal is level with the top of the cone.
- **Step 7:** Dump the contents of the cone into the measuring cup and measure it.

Record your measurement in the Cone column of the chart below.

Step 8: Fill the can or mug to the top edge with cereal. Make sure the top of the cereal is level with the top of the can.

Dump the contents of the can into the measuring cup and measure it.

Record your measurement in the Can or Mug column of the chart below.

	Cone A	Can or Mug B	$\frac{\text{Ratio}}{\text{Cone}} = \frac{\text{A}}{\text{B}}$	Possible Fraction
Your results				
Sample 1	105	300		
Sample 2	150	425		
Sample 3	90	275		

Analysis

1. For each of the four results (yours and the three samples), use a calculator to divide the measurement for the cone by the measurement for the can or mug.

Record the results in the Ratio $\frac{\text{Cone}}{\text{Can}}$ column of the table.

2. For each of the four results, suggest a fraction that represents the ratio of the volume of the cone to the volume of the can.

Record these in the Possible Fraction column of the table.

3. Decide on a single fraction that best compares the relationship between the volume of the cone and can.

Complete the following statement by filling in the blank with a fraction:

The volume of a cone is ______ the volume of a cylinder with equal height and radius.

My Notes

My Notes

Math Lab Part 2: Volume of a Sphere

The volume of a sphere is related to the volume of a cylinder. Do the short investigation below to see how the volumes of a sphere and a cylinder with the same radius are related.

Purpose

To compare the volume of a sphere to the volume of a cylinder.

Materials

- tennis or other small ball (i.e., a sphere)
- empty juice concentrate container or mug (i.e., a cylinder) that is just big enough for the ball
- tape (if using a mug)
- water
- measuring cup or graduated cylinder
- ruler
- felt pen

Procedure

- Step 1: Place the tennis ball inside the empty container or mug. Find the position of the top of the tennis ball while resting inside the container. Mark this position on the outside of the container. If you're using a mug, put a small piece of tape on the outside of the mug and put the mark on the tape if necessary.
- Step 2: Remove the ball from the container.
- **Step 3:** Fill the empty container with water up to the mark on the side. Measure the volume of water that is used by pouring it into the measuring cup. Record the amount in the chart in Analysis. After recording the amount, empty the measuring cup.
- **Step 4:** Place the tennis ball into the empty container. Now fill the container with water up to the mark on the side (make sure that the ball doesn't float). Remove the ball, and then measure the water that was in the container by pouring it into the measuring cup.

	Water in container (mL) A	Water left in container after ball B	Difference Volume taken up by ball A – B = C	Ratio of Ball to water in Container <u>C</u> A	Possible Fraction
Your data					
Sample 1	170	60			
Sample 2	120	45			
Sample 3	300	100			

Analysis

- 1. For each of the four results (yours and the three samples), use a calculator to subtract the amount of water left in the container after the ball was removed (B) from the amount of water that the container can hold (A). The difference gives you the volume taken up by the ball (C).
- 2. For each of the four results, use a calculator to divide the ball measurement (C) by the amount of water that the container can hold (A).
- 3. For each of the four results, suggest a fraction that represents the ratio of the volume of the sphere to the volume of the cylinder $(\frac{C}{A})$.

Record these in the Possible Fraction column of the table.

4. Decide on a single fraction that best compares the relationship between the volume of the cone and can.

Complete the following statement by filling in the blank with a fraction:

The volume of a sphere is ______ the volume of a cylinder with equal radius.

My Notes

Section 2, Lesson B: Discover Math Lab Part 1: Volume of a Cone

	Cone A	Can or Mug B	Ratio	Possible Fraction
Your results				
Sample 1	105	300		
Sample 2	150	425		
Sample 3	90	275		

Section 2, Lesson B: Math Lab Part 2: Volume of a Sphere

Analysis Chart

	Water in container (mL) A	Water left in container after ball B	Difference Volume taken up by ball A – B = C	Ratio of Ball to water in Container CA	Possible Fraction
Your data					
Sample 1	170	60			
Sample 2	120	45			
Sample 3	300	100			
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Module 1, Section 2—Lesson C: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
In what situations might you need to find the surface area and volume of composite 3-D objects?			
How can you find the surface area and volume of composite 3-D objects?			

Section Assignment 1.2 Part 1 Surface Area of 3-D Objects

Find the surface area of the following objects. (12 marks)





3.





5.



6.



Section Assignment 1.2 Part 2 Volume of 3-D Objects

1. Find the volume of this cylinder. (2 marks)



2. Find the volume of this triangular prism. (2 marks)



3. A spherical space vehicle has a radius of 21.0 m. What is the volume of the space vehicle? (2 marks)



4. The shaded portion of the cone is called frustum. Find its volume. (2 marks)



Section Assignment 1.2 Part 3 Surface Area and Volume Problem Solving

1. The curved surface is composed of steel panels 2.5 m by 1.5 m. About how many panels are needed for the roof? (2 marks)



2. This inflatable plastic structure protects construction projects and workers from bad weather. The top is a hemisphere. What volume of space is enclosed by the structure? (2 marks)



3. Each of the four sides of the tent is 3.0 m long. Each side wall is 2.0 m high and the center of the roof is 3.0 m above the ground. Find the volume of the tent. (2 marks)



4. A rectangular prism has length 18.0 cm and width 15.0 cm. Its volume is 4590 cm³. Find its height. (2 marks)



 A cone and a cube have these given dimensions: Cone: 42 cm high, diameter 21 cm Cube: side lengths 0.15 m

Which object has the greater surface area? (2 marks)



6. Find the volume of this solid. (2 marks)

15

15



Section Assignment 1.2 Part 4 Glossary

Write a short definition from your personal glossary for each term below. (1 mark each; 10 marks)

- apex
- lateral area
- right rectangular prism
- right pyramid
- right cone
- right cylinder
- sphere
- surface area
- composite object
- hemisphere

Section Assignment 1.2 Part 5 Multiple Choice

20 marks: 2 marks each

No calculator may be used for this part of the section assignment.

1. Which two objects have the same volume?



- a. I and II
- b. I and III
- c. II and III
- d. The volume is different for each object.

2. Which of the following net diagrams best constructs the square pyramid below?







d.





3. Which of the following shapes has a volume three times the cone shown below?



4. Which net best constructs the figure shown below?













You may use your calculator for questions 5–10.

- 5. A right rectangular prism 3 inches wide, 4 inches long and 10 inches high is half full of water. A large glass marble with a diameter of 2 inches is dropped into the water. How far does the water level rise?
 - a. Less than $\frac{1}{2}$ inches b. Between $\frac{1}{2}$ in and 1 inches
 - c. Between 1 inches and 2 inches
 - d. Over 2 inches
- 6. The volume of the object below is 3636 cm^3 . Calculate the length of *x*.



- a. 17 cm
- b. 15 cm
- c. 16 cm
- d. 14 cm

7. Calculate the surface area of the solid hemisphere shown below.



- a. 622 cm²
- b. 498 cm²
- c. 995 cm²
- d. 747 cm^2
- 8. Calculate the volume of the object below.



- a. 41.8 m³
- b. 29.8 m³
- c. 32.6 m³
- d. 31.1 m³

- 9. A traffic cone is 53 cm high and has a radius of 18 cm. What volume of sand can be packed inside it?
 - a. 4 496 cm³
 - b. 24 429 cm³
 - c. 17 982 cm³
 - d. 53 947 cm³
- 10. The volume of the cube shown below is 59 319 cm³. What is the length of the line AB?



- a. 67.5 cm
- b. 39 cm
- c. 55.1 cm
- d. 78 cm

Title	Marks
Part 1: Surface Area of 3-D Objects	/12
Part 2: Volume of 3-D Objects	/8
Part 3: Surface Area and Volume Problem Solving	/12
Part 4: Glossary	/10
Part 5: Multiple Choice	/20
Total Marks	/62

Foundations of Mathematics and Pre-calculus 10

Module 1, Section 3—Lesson A: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
When can you use trigonometry to solve problems?			
What are the sine, cosine, and tangent ratios?			

S

Ο

Н

С

А

Н

Т

Ο

A

opposite

hypotenuse

adjacent

hypotenuse

adjacent

 $\tan \theta = \frac{\text{opposite}}{2}$

 $\sin\theta = \frac{1}{2}$

 $\cos\theta =$

Were you close?

Sine

Cosine

Tangent

My Notes

Ratio	Description	Colculation	Maamaani
Name	Description	Calculation	winemonic

The ratio of the length of the

The ratio of the length of the

side adjacent to the reference

The ratio of the length of the

side adjacent to the reference

side opposite the reference

angle to the length of the

angle to the length of the

side opposite the reference

angle to the length of the

hypotenuse.

hypotenuse.

	angle.
*A way to	o remember

A great way to remember the trigonometric ratios is by using the following:

SOH CAH TOA

If you chant SOH CAH TOA about 1000 times, it will stay in your head. (It sounds like "so-ka-toe-ah!") Here's what it means:

$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$\cos x = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\tan x = \frac{\text{opposite}}{\text{adjacent}}$$

Notice that the ratios are abbreviated sin, cos and tan.

My Notes

Tip

When you find the sine, cosine or tangent of an angle using your calculator, do not round the result unless it is the final answer to a question. If you need to use this value in subsequent steps, wait until you have reached the final answer before rounding.

Look through the following examples that show you how to use trigonometric ratios to find the measures of the lengths of sides or angles in a right triangle. We'll start with finding the lengths of sides, and then move on to finding angles.

Find a Side Length Using Sine



Find a Side Length Using Cosine

Determine the measure of length *a* to the nearest tenth.



Solution

Label the triangle.



SOH(CAH)TOA

Given: one angle and the adjacent side

Need: the hypotenuse

Solve with cosine.

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$\cos 23^\circ = \frac{a}{8.6 \text{ cm}}$$
$$(8.6 \text{ cm})(\cos 23^\circ) = \left(\frac{a}{8.6 \text{ cm}}\right)(8.6 \text{ cm})$$
$$(8.6 \text{ cm})(\cos 23^\circ) = a$$
$$a = 7.9 \text{ cm}$$

My Notes



you get the same answer that you see here!

$$\cos^{-1}(0.5) = 60$$

Find an Angle Using Sine

Determine the measure of angle A to the nearest degree.

My Notes





Find an Angle Using Tangent

Determine the measure of angle Z to the nearest degree.



My Notes

My Notes

Discover

Math Lab: Comparing Ratios of Sides in Right Triangles

The triangle shown below has one angle that measures 30° (\angle A) and one right angle (\angle C).

Answer the questions about the triangle.



- 3. Calculate the following ratios. Round to four decimal places.
 - a. $\frac{a}{b}$ b. $\frac{a}{c}$ c. $\frac{b}{c}$

Now construct a right triangle DEF of your own with a 30° angle in the space provided. It should look similar to the triangle you just measured, but make it bigger or smaller than the first one. Make sure that the 30° angle is at D and the right angle is at F.

Label your triangle DEF like the following:



- 4. Measure the lengths of the three sides of the triangle.
 - a. Side *d*: _____
 - b. Side *e*: _____
 - c. Side *f*: _____
- 5. Calculate the following ratios. Round to four decimal places.
 - a. $\frac{d}{e}$ b. $\frac{d}{f}$ c. $\frac{e}{f}$

Compare these to the results of the ratios that you calculated in question 3. What do you notice?

My Notes

Foundations of Mathematics and Pre-calculus 10

Module 1, Section 3—Lesson B: Essential Questions

Essential Questions	Before the Lesson: What I Know	After the Lesson: What I Learned	Examples
How do you know whether to use sine, cosine, or tangent when trying to find the unknown measures of a right triangle?			
How is trigonometry used to determine heights and distances that cannot be directly measured?			

Section Assignment 1.3 Part 1 Introduction to Trigonometry

Solve each right triangle. Show all your work. Report your answers to the nearest tenth. (15 marks; 5 marks each)





∠M = _____ KL = _____

KM =_____

Your Name ____

Student No. __



Section 3 Assignment: Math Lab: Clinometer

In this lab you'll create and use a clinometer. A clinometer is an instrument that measures the angle between the ground or the observer and a tall object, such as a tree or building.

You can work with a partner if you like. He or she can help you with some of the measurements.

Materials

- 1 plastic protractor (or use the one from the Appendix)
- cardboard, about 20 cm \times 20 cm (only if you use the paper protractor from the Appendix)
- 1 soda straw, or an 8 ½" \times 11" piece of paper rolled into a narrow tube and taped
- 1 paper clip
- 1 toothpick or another paper clip
- 1 6-8 inch (15–20 cm.) length of thread
- tape (transparent or masking)

Assembling the Clinometer

Your assembled clinometer is shown here. Use the illustration as a guide while following the steps outlined.

Procedure

- **Step 1:** Use tape to attach the straw or the narrow paper tube along the base of the protractor. If you're using the protractor from the Appendix, tape or glue it onto a sturdy piece of cardboard and cut it out first before attaching the straw or paper tube.
- **Step 2:** Tie a paper clip to one end of the piece of thread.
- **Step 3:** Push the other end of the thread through the hole in the centre of the base of the protractor.
- **Step 4:** Tie this end of thread to a toothpick or paper clip.

SECTION 3 ASSIGNMENT



Does your clinometer resemble the one in the illustration above? If so, you're now ready to use your clinometer. Follow the procedure that follows for using the clinometer to measure the height of a tall structure.



Photo by Robert Kneschke © 2010

SECTION 3 ASSIGNMENT

Your Nar	ne Student No
Step 5:	Go outdoors and find a tall structure whose height you can measure. This could be a building, tree, flagpole, or even your own house.
Step 6:	Pick a spot away from the structure where you can see the top of the structure. Mark the spot.
Step 7:	Measure the distance from the base of the structure to the spot you marked. Record this in the table at the end of Step 9.
Step 8:	Stand on the spot and look at the top of the structure through one end of the straw that is attached to your clinometer.
Step 9:	Note the angle at which the string is hanging by putting your finger on the string as you're looking through the straw or tube. Report all your measurements in the table: (4 marks)

Object	Horizontal distance to	Angle measurement	Height from ground
	object	on clinometer	to viewer's eyes

Return to the lesson now, and you'll be directed to finish the math lab after doing some further work with clinometer measurements.

Analysis



Your Name ______ Student No. _____

- 1. What angle on the protractor does the string pass through when the soda straw or paper tube is parallel to the ground? (1 mark)
- 2. If the string passes through 60° when you look through the straw at the top of the tree, then what is the measure of angle *x* in the diagram? (1 mark)
- 3. In the diagram shown, what parts of the triangle will you know after following the procedure above? (2 marks)

4. What trigonometric ratio would you set up to solve for the height of the tree? (1 mark)

5. What other measurement do you need to know in order to determine the height of the tree? (1 mark)

Your Name _____ Student No. _____

Analyzing Your Data

Using the data you recorded at the end of the steps in Procedure, calculate the height of your object.

1. Describe your object or structure. (2 marks)

2. Re-record your measurements. (4 marks)

Object	Horizontal distance to	Angle measurement	Height from ground
	object	on clinometer	to viewer's eyes

3. Provide a sketch of the problem. Include the object, the viewer, and the triangle that is created. Label the measurement of the base and the angle of elevation. (5 marks)

Your Name		 Student No.	

4. Show your calculations for the height of the object. (3 marks)

5. Give the final answer for the height of the object and provide an assessment of how reasonable it is. (3 marks)

Section Assignment 1.3 Math Lab: Clinometer

In this lab you'll create and use a clinometer. A clinometer is an instrument that measures the angle between the ground or the observer and a tall object, such as a tree or building.

You can work with a partner if you like. He or she can help you with some of the measurements.

Materials

- 1 plastic protractor (or use the one from the Appendix)
- cardboard, about 20 cm \times 20 cm (only if you use the paper protractor from the Appendix)
- 1 soda straw, or an 8 $\frac{1}{2}$ " \times 11" piece of paper rolled into a narrow tube and taped
- 1 paper clip
- 1 toothpick or another paper clip
- 1 6-8 inch (15–20 cm.) length of thread
- tape (transparent or masking)

Assembling the Clinometer

Your assembled clinometer is shown here. Use the illustration as a guide while following the steps outlined.

Procedure

- Step 1: Use tape to attach the straw or the narrow paper tube along the base of the protractor. If you're using the protractor from the Appendix, tape or glue it onto a sturdy piece of cardboard and cut it out first before attaching the straw or paper tube.
- Step 2: Tie a paper clip to one end of the piece of thread.

Step 3: Push the other end of the thread through the hole in the centre of the base of the protractor.

Step 4: Tie this end of thread to a toothpick or paper clip.


Does your clinometer resemble the one in the illustration above? If so, you're now ready to use your clinometer. Follow the procedure that follows for using the clinometer to measure the height of a tall structure.



- Step 5: Go outdoors and find a tall structure whose height you can measure. This could be a building, tree, flagpole, or even your own house.
- Step 6: Pick a spot away from the structure where you can see the top of the structure. Mark the spot.
- Step 7: Measure the distance from the base of the structure to the spot you marked. Record this in the table at the end of Step 9.
- Step 8: Stand on the spot and look at the top of the structure through one end of the straw that is attached to your clinometer.
- Step 9: Note the angle at which the string is hanging by putting your finger on the string as you're looking through the straw or tube. Report all your measurements in the table:(4 marks)

Object	Horizontal distance to object	Angle measurement on clinometer	Height from ground to viewer's eyes

Return to the lesson now, and you'll be directed to finish the math lab after doing some further work with clinometer measurements.

Analysis



- 1. What angle on the protractor does the string pass through when the soda straw or paper tube is parallel to the ground? (1 mark)
- 2. If the string passes through 60° when you look through the straw at the top of the tree, then what is the measure of angle x in the diagram? (1 mark)
- 3. In the diagram shown, what parts of the triangle will you know after following the procedure above? (2 marks)

- 4. What trigonometric ratio would you set up to solve for the height of the tree? (1 mark)
- 5. What other measurement do you need to know in order to determine the height of the tree? (1 mark)

Analyzing Your Data

Using the data you recorded at the end of the steps in Procedure, calculate the height of your object.

1. Describe your object or structure. (2 marks)

2. Re-record your measurements. (4 marks)

Object	Horizontal distance to object	Angle measurement on clinometer	Height from ground to viewer's eyes

Provide a sketch of the problem. Include the object, the viewer, and the triangle that is created. Label the measurement of the base and the angle of elevation.
 (5 marks)

4. Show your calculations for the height of the object. (3 marks)

5. Give the final answer for the height of the object and provide an assessment of how reasonable it is. (3 marks)

Section Assignment 1.3 Part 2 Solving Right Triangle Problems

Answer the following questions to the nearest tenth.

1. A guy wire must be attached to telephone pole 8.2 meters above the ground and should make an angle of 50° with the ground. How long should the wire be? (2 marks)

2. From the top of the CN Tower, the angle of depression to the tip of the tower's shadow is 88°. The shadow is 19.5 m long. How tall is the CN Tower? (2 marks)

Two buildings are 31.7 m apart. From the 12th floor of the shorter building, the angle of elevation to the top of the taller building is 27°. The angle of depression to the base of the taller building is 48°. What is the height of the taller building? (5 marks)

Two tracking stations, A and B, measure the height of a rocket to be 4 km. The angles of elevation of the rocket are found to be 72° and 55°. How far apart are the stations A and B? (5 marks)



Section Assignment 1.3 Part 3 Glossary

Write a short definition from your personal glossary for each term below. (1 mark each; 11 marks)

- adjacent side
- cosine ratio
- hypotenuse
- opposite side
- reference angle
- sine ratio
- solving a triangle
- tangent ratio
- angle of depression
- angle of elevation
- clinometer

Section Assignment 1.3 Part 4 Multiple Choice

20 marks: 2 marks each

No calculator may be used for this part of the section assignment.

1. Determine the ratio of sin A.



What are the steps to solve for the length of *x*?

- a. Use the Pythagorean Theorem.
- b. Use cos 16° to solve for the common side, then cos 24°.
- c. Use tan 16° to solve the common side, then cos 40°.
- d. There is not enough information given to solve for *x*.

3. Jen is helping to build a wheelchair ramp that will have a 6% grade (i.e., a rise of 6 cm for a horizontal change of 100 cm). Which of the following expressions will calculate the angle between the ground and the top part of the ramp?



a.
$$\cos = \left(\frac{6}{100}\right)$$

b.
$$\tan^{-1} = \left(\frac{6}{100}\right)$$

c.
$$\cos^{-1} = \left(\frac{6}{100}\right)$$

d.
$$\tan = \left(\frac{6}{100}\right)$$

You may use your calculator for the last six questions.

4. Using a protractor, measure one of the unknown angles and determine the unknown length *n*.



- a. 8.4 m
- b. 1.5 m
- c. 1.3 m
- d. 3.2 m
- 5. In \triangle DEF, \angle D = 90°, DE = 5 m, and EF = 13 m. Calculate the measure of \angle EFD.
 - a. 69°
 - b. 21°
 - c. 67°
 - d. 23°

6. Katie is making plans to build a skateboard ramp, and she needs to know how much wood to buy. Using the diagram below, calculate the area of the top part of the ramp.



- a. 25.6 square feet
- b. 102.3 square feet
- c. 50.5 square feet
- d. 51.1 square feet
- 7. A 9 metre tall house is located 14 m away from a tree. The angle of elevation from the roof of the house to the tree is 25°. If the tree falls directly toward the house, which of the following will occur?



- a. The top of the tree will just touch the side of the house.
- b. The tree will not hit the side of the house.
- c. The tree will hit the side of the house.
- d. There is not enough information to tell what will happen.

A surveyor needs to know the height of a cliff on the other side of a stream. To calculate this, he positions his assistant 27 m away on the same side of the stream, and then measures the angles as shown below. What is the height of the cliff?



- a. 46.4 m
- b. 62.9 m
- c. 87.5 m
- d. 29.9 m
- 9. Alex is at the top of a 42 m tower. He spots two cars on the roads below, one due west of the tower and one due south. The angle of depression to the car in the south is 21°, and the angle of depression to the car in the west is 10°. How far are the cars away from each other?



- a. 275 m
- b. 62 m
- c. 269 m
- d. 262 m

10. What are the sine, cosine, and tangent for angle x?



Title	Marks
Part 1: Introduction to Trigonometry	/15
Math Lab: Clinometer	/27
Part 2: Solving Right Triangle Problems	/14
Part 3: Glossary	/11
Part 4: Multiple Choice	/20
Total Marks	/87



UNIT CONVERSION

	Common Imperial	Imperial and Metric	Metric
Length	1 mile = 1760 yards	1 mile ≈ 1.609 km	1 km = 1000 m
	1 mile = 5280 feet	1 yard ≈ 0.9144 m	1 m = 100 cm
	1 yard = 3 feet	1 foot ≈ 0.3048 m	1 cm = 10 mm
	1 yard = 36 inches	1 inch ≈ 2.54 cm	
	1 foot = 12 inches		
Mass	1 ton = 2000 pounds	1 pound ≈ 0.454 kg	1 t = 1000 kg
(Weight)	1 pound = 16 ounces	1 ounce ≈ 28.35 g	1 kg = 1000 g
Common	mile = mi		kilometre = km
Abbrevia- tions	yard = yd		metre = m
	ton = ton		centimetre $=$ cm
	feet = ' or ft		millimetre = mm
	inch = " or in		tonne (metric ton) = t
	pound = lb		gram = g
	ounce = oz		

FORMULAE







GEOMETRIC FORMULAE

Key Legend		
l = length	P = perimeter	
w = width	C = circumference	
b = base	A = area	
h = height	SA = surface area	
s = slant height	V = volume	
r = radius		
d = diameter		

Geometric Figure	Perimeter	Area
Rectangle	P = 2l + 2w	
	or	A = lw
	P = 2(l+w)	
Triangle		
	P = a + b + c	$A = \frac{bh}{2}$
Circle	$C = \pi d$	
r	or	$A = \pi r^2$
d	$C = 2\pi r$	

NOTE: Use the value of π programmed in your calculator rather than the approximation of 3.14.



Geometric Figure	Surface Area	Volume
Cylinder	$A_{top} = \pi r^{2}$ $A_{base} = \pi r^{2}$ $A_{side} = 2\pi rh$ $SA = 2\pi r^{2} + 2\pi rh$	$V = (area of base) \times h$
Sphere	$SA = 4\pi r^2$ or $SA = \pi d^2$	$V = \frac{4}{3}\pi r^3$
Cone	$A_{side} = \pi rs$ $A_{base} = \pi r^{2}$ $SA = \pi r^{2} + \pi rs$	$V = \frac{1}{3} \times (\text{area of base}) \times h$
Square-Based Pyramid	$A_{triangle} = \frac{1}{2}bs \text{ (for each triangle)}$ $A_{base} = b^2$ $SA = 2bs + b^2$	$V = \frac{1}{3} \times (\text{area of base}) \times h$
Rectangular Prism	SA = wh + wh + lw + lw + lh + lh or SA = 2(wh + lw + lh)	$V = (area of base) \times h$
General Right Prism	SA = the sum of the areas of all the faces	$V = (area of base) \times h$
General Pyramid	SA = the sum of the areas of all the faces	$V = \frac{1}{3} \times (\text{area of base}) \times h$

