Let’s Communicate and Represent Math Using Trigonometry!

Pre-calculus 12

Suggested time: 60 minutes

Purpose

To provide students with an opportunity to develop and assess their communicating and representing skills.

Core Competency: Communication

Curricular Competency Organizer: Communicating and Representing

- Explain and justify mathematical ideas and decisions in many ways
- Represent mathematical ideas in concrete, pictorial, and symbolic forms
- Use mathematical vocabulary and language to contribute to discussions in the classroom
- Take risks when offering ideas in classroom discourse

Course Content

Pre-calculus 12 Trigonometric functions*

*Course content can be adapted to the grade level and the topic at hand. For example, this activity can be used with Math 10 students for solving trigonometry problems involving trigonometric ratios. The contextualized problems will need to be adjusted to suit the selected course. This activity can also be applied to Math 8 for exploring the Pythagorean relationship.

Rationale

This activity is intended to highlight the value of communicating and representing within mathematics. To inform students of their current abilities in mathematical communicating and representing, students will be self-assessing at the end of an activity that provokes communication and representing. This self-assessment is completed regularly over time, so students can identify areas that need growth and celebrate successes in this Curricular Competency. The activity is designed around Peter Liljedahl’s Thinking Classroom model that has students working collaboratively at vertical non-permanent surfaces through rich tasks. In working collaboratively, students can think about their own communicating with others in

their groups. The vertical non-permanent surfaces support students in taking chances and risks when working on mathematics. Student groups are formed randomly (Liljedahl, 2015).

Videos on this activity's design considerations, classroom delivery and teacher/student reflections are available on Focusing on Competencies in Math.

Core Competency Elaboration

- Connecting and engaging with others
- Acquiring and presenting information
- Working collectively
- Supporting group interactions

Big Ideas

- Understanding the characteristics of families of functions allows us to model and understand relationships and to build connections between classes of functions.
- Transformations of shapes extend to functions and relations in all of their representations.

Learning Standards

- Explain and justify
    - Use mathematical arguments to convince
    - Includes anticipating consequences
    - Decisions
    - Have students explore which of two scenarios they would choose and then defend their choice
    - Many ways
    - Including oral, written, visual, use of technology
    - Represent
    - Using models, tables, graphs, words, numbers, symbols
    - Connecting meanings among various representations
    - Discussions
    - Partner talks, small-group discussions, teacher-student conferences
    - Discourse
    - Is valuable for deepening understanding of concepts
    - Can help clarify students' thinking, even if they are not sure about an idea or have misconception

Content Elaborations

- Trigonometry
    - Graphing primary trigonometric functions, including transformations and characteristics
    - Solving problems in situational contexts

First Peoples Connections

The First Peoples Principles of Learning that are involved in working through the problems are:

- Learning is holistic, reflexive, reflective, experiential, and relational.
- Learning is embedded in memory, history, and story.
- Learning involves patience and time.
- Learning requires exploration of one's identity.

The Changing Temperature and the Tidal Waves questions connect with local First Peoples community practices, as detailed in the understanding and solving competency:

- Engage in problem-solving experiences connected with place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures

In both sample questions, students are asked to connect to a situation relevant to a local First Nations community. They need to imagine their life in nature by reflecting upon their past experiences and memory of what it would be like gathering berries in the wild or paddling a canoe in tidal waters. While solving the problem, students need to exercise patience and perseverance to understand and find different ways to approach the question.

- Materials
    - Whiteboards or whiteboard wall and whiteboard markers
    - A set of contextualized problems
    - Alternatively, windows and window writers or whiteboard markers.
    - Optional: Graphing calculator

Vertical Non-Permanent Surface Group Activity

We prefer to have students working at vertical non-permanent surfaces for this task for a variety of reasons. If you don't have enough white boards, windows

work too. You can buy window markers or use white board markers. When working at a vertical surface, the students' work is public, which is a benefit for the teacher and for the students. The teacher can see which groups are stuck or might need some hints to progress. Students often get to work faster when they are writing up on the white board. Students see what other groups are doing and use this work as inspiration for their work, which is a more efficient mode of knowledge transfer. The non-permanence of the surface provides a safe space for students to take chances and make mistakes. This activity improves student engagement and participation when working towards a solution (Liljedahl, 2019).

Overview

This activity takes place at the end of a Trigonometric Functions Unit. Students should be proficient in the transformations of trigonometric functions and modeling situations with these functions.

In this activity, students are placed into random groups of three or two, if necessary. Random groups are preferred as this provides students with the opportunity to collaborate with different students each time.

At the beginning, students are informed that the learning focus is Communicating and Representing. They are provided with the assessment rubric (see Appendix A) for discussion before the activity begins.

Student Self-Assessment

Students complete the self-assessment (see Appendix A) at the end of the activity. The rubric is designed by describing the activity straight from the Curricular Competency in the curriculum document. The self-reflection portion is divided into two sections. The first part is for students to reflect on and set goals for future work in the communicating and representing competency (see Appendix A). The second portion is for students to reflect on and set goals for future work in the communication Core Competency (see Appendix B).

Grading or evaluation is intentionally absent from the template, as it can negatively impact student honesty and sincerity in completing this assessment. The primary purpose behind this assessment is to provide an opportunity for students to reflect on their own competency around communicating and representing and to give value to this in a classroom setting. The value comes through in the time devoted to the activity itself, and also in how the teacher decides to include these assessments in the reporting of student progress.

Formative Assessment for the Learning Activity

The formative assessment for the learning activity (see Appendix C) is descriptive feedback in conjunction with the strength-based language. There are criteria for assessment for the course content and a separate assessment with a focus on the Curricular Competency. Similar to the Student Self-Assessment, there is a self-reflection piece with regards to the content and the Curricular Competency where students identify where they are at with their learning, areas they would like to see improvement, and goal setting for next steps. The formative assessment includes a checklist for students to reflect on and set goals for future work on the embedded Core Competencies.

Student-Teacher Choice: Contextualized problems

1. Length of Day: Time and Date
2. Salmon Run: Life Cycle of Salmon
3. Tidal Waves*: Tide Times for Deep Cove
4. Sound Travel*: Understanding Sound Waves
5. Pendulum: Simple Harmonic Motion
6. Weighted Spring Interactive Simulation
7. Changing Temperature* Chilliwak Climate
8. Wheels on the Bus: Rotating Wheel
9. Ferris Wheel: Ferris Wheel Trigonometry Problem

*These specific topics are included in the sample questions.

Sample Questions

Changing Temperature

Chilliwack Climate

The following table shows weather by month and temperature/precipitation averages in Chilliwack, BC:

	January	February	March	April	May	June	July	August	September	October	November	December
Avg. Temperature (°C)	1.7	3.8	6.6	9.5	12.9	15.5	17.8	17.7	15	10.4	5.9	2.8
Min. Temperature (°C)	-1	0.4	2.4	4.7	7.7	10.4	11.8	11.7	9.5	6.2	2.9	0.2
Max. Temperature (°C)	4.5	7.3	10.8	14.4	18.1	20.6	23.8	23.7	20.5	14.7	8.9	5.5
Avg. Temperature (°F)	35.1	38.8	43.9	49.1	55.2	59.9	64.0	63.9	59.0	50.7	42.6	37.0
Min. Temperature (°F)	30.2	32.7	36.3	40.5	45.9	50.7	53.2	53.1	49.1	43.2	37.2	32.4
Max. Temperature (°F)	40.1	45.1	51.4	57.9	64.6	69.1	74.8	74.7	68.9	58.5	48.0	41.9
Precipitation / Rainfall (mm)	215	165	147	118	95	78	54	55	97	168	221	227

For thousands of years, First Nations peoples have gathered information about the area they live in so that they know the best times, places, and ways to grow, tend and harvest their nutrient-rich foods.

Berry patches are tended and protected so important nutrients (that we now call antioxidants, vitamin C, etc.) are available to the whole community, ensuring community health.

You are part of a small group of young people being apprenticed by the women who carry the knowledge of where the berries and other important foods grow, how to tend the patches so they continue to produce crops that can feed the whole community, and the ways to best harvest them.

You learn that berries start ripening when the temperature is comfortable when no extra layer of clothing is required to keep you warm. Create two trigonometric models, one for average temperature and one for rainfall. Use these models to determine a range of dates that might be best for berry-gathering. Include any assumptions you have made.

Possible Extensions: Develop another sinusoidal function to best approximate the optimal times in a day for berry-gathering. Use the data from Time and Date. Justify your solution with more than one method.

Tidal Waves

Tide Times for Deep Cove

The following is a table that shows high tide and low tide on Friday, March 8 and Saturday, March 9 in Deep Cove:

DEEP COVE TIDE TABLE:

Friday 8 March	6:39 AM	PST			Sunrise
	7:36 AM	PST	4.41 m	**(14.46 feet)**	**High Tide**
	7:54 AM	PST			Moonrise
	1:10 PM	PST	2.34 m	**(7.67 feet)**	**Low Tide**
	6:05 PM	PST			Sunset
	7:29 PM	PST	4.08 m	**(13.39 feet)**	**High Tide**
	8:20 PM	PST			Moonset
Saturday 9 March	**1:12 AM**	PST	1.95 m	**(6.39 feet)**	**Low Tide**
	6:38 AM	PST			Sunrise
	7:55 AM	PST	4.35 m	**(14.26 feet)**	**High Tide**
	8:15 AM	PST			Moonrise
	1:45 PM	PST	2.13 m	**(6.99 feet)**	**Low Tide**
	6:07 PM	PST			Sunset
	8:15 PM	PST	4.06 m	**(13.33 feet)**	**High Tide**
	9:26 PM	PST			Moonset

You are paddling a canoe from Deep Cove to Say Nuth Khaw Yum Provincial park near the end of Indian Arm. When the tide is falling, the current in the Arm is extremely challenging to paddle. Use the data to create a trigonometric model and estimate the best time to leave on this journey. The distance for this trip is about 15 km, and canoes can travel about 5 km/h.

Say Nuth Khaw Yum means “Serpent’s Land.” It is in the core of Tsleil-Waututh traditional territory that has, from time immemorial, been an area of significance to the Tsleil-Waututh people. For over a millennium, the Tsleil-Waututh Nation has continued to use the land, water, and resources of the entire Indian Arm.

Possible Extensions

According to the provided data, provide another sinusoidal function to approximate the worst time to travel from Deep Cove to Say Nuth Khaw Yum Provincial Park. Justify your solution with more than one method.

- Suppose you want to stay at the Park for three hours before returning to Deep Cove. Is this feasible with the currents? Why or why not?

Sound Travel

Understanding Sound Waves

Sound is a compression wave propagating through a medium made of atoms and molecules. Frequency is the number of cycles a molecule travels per second, measured in hertz (Hz). Frequency is inversely related to the period which is the time it takes for one cycle of the wave to pass. The frequency of sound is commonly known as pitch. The faster the wave of moving molecules, the higher the frequency, consequently, the higher the pitch.

The loudness of the sound is defined by the distance a molecule shifts from the equilibrium due to the amount of energy transfer. This displacement of the molecule leads to sound pressure in the air. This displacement of a molecule is measured in decibels (dB). The greater the energy transfer, the greater the displacement of the molecule from equilibrium, and the louder the sound is.

Watch the YouTube clip on Understanding Sound Waves, determine the sinusoidal functions that represent the normal hearing range of a person.

Possible Extensions

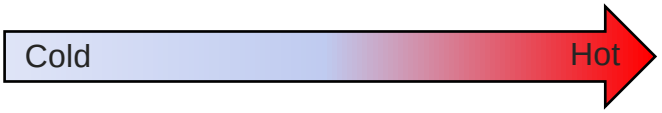
Compare and contrast the sinusoidal functions that represent the hearing ranges of a beluga whale and a bat. How do these animals' hearing range compare to the humans' hearing range? Why do you suppose that is? What are some of the implications of different mammals' hearing ranges?

Writer's Reflection

Students are encouraged to use technology to ensure that the trigonometric models accurately represent the given data. How do the students know if their solution is correct? What are some ways to check their answer? Some of my own students graphed their functions and matched the tables of values from their graphing calculator to the original sets of data. Students who know how to model a function using scatter plot function may use it to check their solution. More advanced students may discover that a single trigonometric function cannot perfectly model the provided data. A combined function or a composite function may be more appropriate for some of the questions.

Appendix A

Communicating and Representing Student Self-Assessment

I am able to …		How am I doing …
My Mathematical Solution	… write a solution that shows my steps and gives reasons for my steps. … provide a variety of methods to my answer.	 Explain:
	… describe my thinking with words, algebra, diagrams, graphs, and/or other forms.	 Explain:
My Collaboration	... use relevant mathematical vocabulary and language when describing my thinking with others in my group.	 Explain:
	… offer my ideas without fear and persevere in communicating my ideas with others.	 Explain:

Curricular Competency Self-Reflection
During this activity, ***what did I do well*** with regards to communicating and representing mathematics? Please provide a specific example(s).
Based on this activity, ***what would I like to improve*** on with regards to communicating and representing mathematics? Please provide a specific example(s).
What can I do to improve my ability to communicate and represent mathematics? Please provide a specific example(s).

Appendix B

Communication Core Competency Student Self-Reflection

Check off All the points that you have met during this activity.
 	
• I can connect and engage with others to share and develop ideas
  ○ I ask and respond to simple, direct questions.
  ○ I am an active listener; I support and encourage the person speaking.
  ○ I recognize that there are different points-of-view, and I can disagree respectfully.
• I can acquire, interpret, and present information
  ○ I can understand and share information about an idea that is important to me.
  ○ I present information in an organized way.
  ○ I can present information and ideas to an audience I may not know.
• I can work collectively to plan, carry out, and review the activity
  ○ I can work with others to achieve a common goal; I do my share.
  ○ I can take on roles and responsibilities in a group.
  ○ I can summarize key ideas and identify the ways we agree.
• I support group interactions
  ○ I give, receive, and act on feedback.
  ○ I can recount the experience and activity and tell something I learned.
  ○ I can represent my learning and tell how it connects to my experience and efforts.

During this activity, ***what did I do well*** with regard to mathematical communication? Please provide a specific example(s).
Based on this activity, ***what would I like to improve*** on with regards to mathematical communication? Please provide a specific example(s).
What can I do to improve my mathematical communication abilities? Please provide a specific example(s).

Appendix C
Formative Assessment for the Learning Activity

Please use a highlighter to indicate the proficiency descriptors that best match your understanding of the course content and the Curricular Competency.

Indicate your level of ability based on the descriptors by highlighting one of the proficiency indicators.

Proficiency Indicators	Course Content:
Emerging	I can understand and recount the objective of the question. I can identify the area of the course content the question is based on.
Developing	I can understand and identify the area of the course content the question is asking. I can come up with an approach to solve the problem.
Proficient	I understand and can identify the objectives of the problem. I can clearly explain and represent different approaches to solve the problem. I am able to arrive at a solution with my group in multiple ways.
Extending	I fully understand and can quickly identify the objectives of the problem. I can clearly explain and represent different approaches to solve the problem. I can lead the group collaboratively to represent the solution in multiple ways, including different ways to verify the solution. Working through the problem causes me to consider other relevant aspects of the question that are not stated in the question.
	Curricular Competency: Communicating and Representing
Emerging (#1, #2 or 1 to 2 out of 7)	1. I can explain and justify using mathematical arguments to convince. 2. I can engage in discussions through small-group discussions. 3. I can represent using models, tables, graphs, words, numbers, and connect meanings among various representations.
Developing (#1 to #4 or	

3 to 4 out of 7) Proficient (#1 to #5 or 5 out of 7) Extending (#1to #7 or 6 to 7 out of 7)	4. I can represent in many ways, such as oral, written, visual, and use of technology. 5. I can make decisions by exploring different scenarios where I can choose and defend my choice. 6. I can explain and justify using mathematical arguments, and I can anticipate the consequences. 7. I can engage in mathematical discourse to deepen my understanding of math concepts, and I can clarify my thinking even if I am not sure about an idea or have misconceptions.
Reflection on the Course Content and the Curricular Competency	
Areas I did well	
Areas I wish to do better	
What steps can I take to improve on the areas I wish to do better?	

Appendix C

Embedded Core Competencies	Checklist	Points
Communication	☐ I can connect and engage with others to share and develop ideas ☐ I can acquire and present information ☐ I can focus on intent and purpose	
Collaboration	☐ I can work collectively ☐ I can determine common purposes ☐ I can support group interactions	
Creative Thinking	☐ I look for new perspectives, new problems, or new approaches. ☐ I can generate creative ideas as a result of engagement with someone else's ideas. ☐ I can further develop my ideas by evaluating and refining my original ideas.	
Critical and Reflective Thinking	☐ I can gather, select, evaluate, and synthesize information. ☐ I can ask questions and offer judgments, conclusions, and interpretations supported by evidence I or others have gathered. ☐ I can examine my thinking, seek feedback, reassess my work, and make adjustments.	

	☐ I can develop or adapt criteria, check information, assess my thinking, and develop reasoned conclusions, judgments, or plans.	
Positive Personal and Cultural Identity	☐ I understand that my characteristics, qualities, strengths, and challenges make me unique and are an important part of the communities in which I belong. ☐ I understand that what I value influences the choices I make and how I present myself in various contexts. ☐ I can explain how I can use my strengths to contribute to my home and/or communities.	
Personal Awareness and Responsibility	☐ I use strategies for working toward a healthy and balanced lifestyle, for dealing with emotional challenges, and for finding peace in stressful times. ☐ I have valuable ideas to share. ☐ I take responsibility for my learning, seeking help as I need it. ☐ I can set priorities, implement, monitor, and adjust a plan; and assess the results.	
Social Awareness and Responsibility	☐ I work to make positive changes in the communities I belong to and the natural environment. ☐ I am aware of how others may feel and take steps to help them feel included. ☐ I can clarify problems or issues, generate multiple strategies, weigh consequences,	

	compromise to meet the needs of others, and evaluate actions. - I use respectful and inclusive language and behavior, including in social media. I can advocate for others.	
Reflection on Embedded Core Competencies		
Areas I did well		
Areas I wish to do better		
What steps can I take to improve on the areas I wish to do better?		

Sample Lesson Plan

Please adjust timing to correspond to the length of your class.

Activity	Teacher	Student
As students enter the classroom	• Puts students into random groups of two to three students	• Gets to know the group members
Opening (5 – 10 minutes)	• Introduces five Core Competencies and four Curricular Competency organizers to the students • Introduces the Core Competency and the curricular organizer students will be practicing	• Names and understands the five Core Competencies • Understands the specific Core Competency and the Curricular Competency organizer
Introduce the Activity (5 minutes)	• Hands out a selection of three-five contextualized problems and the student self-assessment • Briefly speaks to each problem • Brings students' attention to the self-assessment to be completed at the end of the activity (Appendix A and B) • Asks the groups to select a problem	• Becomes familiar with the provided problems • Becomes familiar with the self-assessment to be completed • Selects a problem to work on as a group based on the topics and the brief description • Focuses on the Core and Curricular Competency as they work through the problem
Before solving the problem (5 minutes)	• Allows students time to read over the problems • Asks students to strategize and find relevant resources for solving the question of their choice	• Reads over the selected question together as a group • Strategizes within the group how to approach the problem

Solving the problem (30 minutes)	• Assigns groups to a vertical non-permanent surface with erasable markers • Circulates the room and observes student collaboration • Answers student questions as they come up throughout the activity	• Works on a vertical non-permanent surface • Explains and justifies their points to their group • Makes decisions about what to do to solve the problem in multiple ways • Represents their solution in many ways • Completes the problem through discussions and discourse
Teacher formative assessment (1 minute)	• Asks groups to digitally document their work and email to teacher or post on class blog/website	• Takes photo(s) of their work and sends the photo(s) to the teacher or upload them to the class blog/website
Student Self-assessment and reflection (5 – 10 minutes)	• Asks students to return to their seats and complete the self-assessment to be handed in at the end of the class	• Completes the self-assessment and reflection and hands it in at the end of the class
Formative Assessment for the Learning Activity (Exit slip or class starter for the following class)	• Hands out the Formative Assessment for the Learning Activity ○ This can be created into an online form for students to complete and for teacher to have instant feedback.	• Completes the formative assessment and returns it to the teacher
Teacher record	• Collects the self-assessments and keep them in a binder to track student progress and development throughout each term	• Revisit their previous self-assessments to reflect upon their growth in the areas of communication and representation throughout the course

Addressing Diverse learning needs

Students with specific learning needs may benefit from seeing the activities and the choice questions ahead of time. Allowing the students to read over and process the activity will help students who have specific needs in reading and comprehension or processing written information. These students may require adaptations such as extra time, separate settings, a reader, a scribe, or one-on-one guidance. Different learning needs require various adaptations. One common adaptation for diverse learners would be to divide up the activity into smaller tasks.

Since the activity is strength-based, depending on the specific learning needs, the student may wish to work on a single area of communication each time. The student may take a similar approach for the communicating and representing competency. Student can work on mastering one aspect of communicating and representing at a time. For example, the student may want to only focus on writing a solution that shows steps and giving reasons for the steps. Next time the teacher can help the student work on describing their thinking with words, algebra, diagrams, graphs, and/or other forms. When it is appropriate, the teacher can exercise their professional autonomy to combine or divide the different areas of communicating and representing competency.

The suggested time for this learning activity is for learners with no diverse learning needs. Thus, this activity may take up to two classes for students who struggle with specific areas of learning.

Teacher Self-Reflection

After the activity, the teacher needs to document the experience and compare the documentation at the end of the term or school year.

How did the activity go? Be specific.	
What will I do differently next time?	

Writer's Reflection

I ran the activity with my students four times, and each time the student did better with their communicating and representing competency. Students' reflections became more specific to the Curricular Competency, and they also were less over-confident with their competency level. By the third time, they realized that this particular competency is something that they can continue to work on throughout the rest of their education. Since I did the activity for proving trigonometric identities, solving logarithmic equations, and solving logarithmic problems, the students realized that different strategies are needed for different topics. The students were challenged each time I ran this activity with different learning standards.

I find the most valuable part of the activity to be the last question in each of the assessments on what the student plans to do in order to make improvement. I observed student growth each time we do the activity by looking at what they answered for that question. The original version of the student self-assessment did not have the third question in the self-reflection. This question was added after seeing how my students responded to the first two questions. The self-reflection appeared incomplete without allowing students to speak to how they would grow and improve on this competency. After adding the last question, students thought deeper about the activity and found value in participating in this activity.

Since this learning activity is to use curricular content as a vehicle to work on a specific Curricular Competency, the focus is the communicating and representing competency. This activity does not answer all the contextual problems presented in the activity. Teachers can exercise their discretion whether they see value in going over or providing the solutions to the problems presented in the activity.

If there is one thing I would do differently, it would be to shift the focus to each of the Curricular Competencies at a time until ultimately, all of the Curricular Competencies are covered. When students are familiar with all the Curricular Competencies within the mathematical context, I would then combine and include reasoning and modelling competency, understand and solving competency, as well as connecting and reflecting competency into student self-assessment and the formative assessment. I did not do so for this learning activity because I do not want to obscure the original purpose and intention of this activity plan which is to explicitly incorporate communicating and representing competency into mathematical content.

References

Liljedahl, P. (2015, Feb. 14). *Building Thinking Classrooms: Conditions for Problem Solving*. Retrieved from http://peterliljedahl.com/wp-content/uploads/Building-Thinking-Classrooms-Feb-14-20151.pdf

Liljedahl, P. (2019, June 25). *Building Thinking Classrooms*. Retrieved from http://www.peterliljedahl.com/publications/building-thinking-classrooms