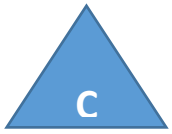
Playing with Quadratics

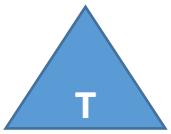
Pre-calculus 11 Topic: Graphing Quadratic Functions

Stage 1 – Desired Results



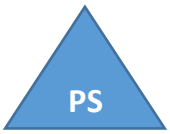
Communication I can…

I can communicate about math concepts in a variety of ways. I can explain my thinking and reasoning to peers and adults with growing confidence.



Thinking I can…

I can build on ideas to create new things. I can analyze information from different perspectives.



Personal and Social I can…

I can persevere with challenging tasks. I can reflect on my strengths and abilities.

STUDENTS WILL UNDERSTAND

Big Idea: Quadratic relationships are prevalent in the world around us.
They can be represented visually, symbolically, and can be described and determined by their vertex, value and number of intercepts, the direction of opening, and point values.

Essential Question:
What effect do the various parameters of a quadratic function have on its graph?

STUDENTS WILL DO

Curricular Competencies:

Understanding and Solving
Students will apply flexible thinking and multiple strategies to solve problems. They will develop, demonstrate, visualize, and apply understanding through play and inquiry.

STUDENTS WILL KNOW

Content:
Graphing Quadratic Functions

Stage 2 – Learning Plan

Resources / Materials Required / Technology

Deal with U! card activity
Card sets, wipe-board (slipcovered graph paper), dry-erase marker, and paper towel for each student

Graphing Aerobics
Projector and computer or equation cards

Online Games
Device for each student or group
Projector and computer (optional)

Differentiation

- Provide a list of vocabulary related to quadratic functions with pictures at the beginning of the unit.
- Allow more/less time for students as needed.
- Offer a variety of groupings (individual, partner, groups of different sizes).
- Offer more or less challenging questions to students. Offer choice. Cards come in varying levels of difficulty.
- Offer time to move through the levels of complexity, to revisit and review.

Notes

- Students get to create, play, move, draw, talk, and engage in many ways of learning and representing through three activities: Deal with U! card game, Graphing Aerobics and Online Games.
- The activities are designed such that teachers can manipulate the instructions to address the desired outcome in terms of difficulty, content, time frame, and student groupings.
- Activities can be used on their own, together, or in a progression. They can be used to introduce or review content and as both formative and summative assessment.
- They could also be adjusted and used as a review before a Pre-calculus 11 test or as a preview for transformations in Pre-calculus 12.

Stage 3 – Assessment Evidence

Formative (choose based on how much time you have available)

- Activities with Deal with U! cards (individual or partner work, see progression list attached)
- Games (graphing aerobics, online quizzes – see more details below)

Summative

- Deal with U! worksheet (see attached file, 2-sided)

Self (Assessment/Reflection)

- See the end of document for the Checklist

Stage 4 – Teaching Plan

Thinking Behind the Thinking

We wanted a variety of ways for students to engage with quadratic functions to deepen their understanding. We wanted teachers to be able to take pieces of the activity and use them in a multitude of different ways. We wanted to incorporate games and movement, as well as various student groupings. The intention was to motivate and engage students in different ways, to help them focus their attention while deepening the understanding of the content, along with the process of learning and understanding the content.

We found initially, the parameters of Deal with U! did not work. We made adjustments and then realized it would be nice to have levels of difficulty. The challenge then was to create a variety of parameters that could work together within different levels, knowing that some parameters might contradict each other. When parameters contradict each other (for example, opening up with a vertex of (1,1) and having two x-intercepts) we had discussions in small groups or with the class about why it did not work. We then asked the question, “What could we change so that this would work?”

We wanted the activities to be flexible for teachers so that they could use them in various groupings. They could use parts of them, they could use them as an introduction, as a review and even as formative or summative assessments. We wanted lots of room, for adaptations and challenges and students being able to work at their own pace.

Videos on this activity’s design considerations, classroom delivery and teacher/student reflections are available on Focusing on Competencies in Math.

Deal with U! card activity

This is a progressive activity going from basic quadratic qualities to advanced where students sketch graphs and write formulas based on given parameters from randomly drawn cards.

The activity can be done as a large class group, with smaller groups of students, in pairs or individually. It can be used to introduce concepts, as well as to reinforce, review and extend learning, as both formative and summative assessment. Teachers may use one or more of the four card sets in one lesson or over several lessons.

To Play

Every group needs a set of cards and every group member needs a surface to draw and write on. This could be a whiteboard, paper, or graph paper. Ideally, a non-permanent surface would be used, for example, graph paper in a plastic slipcover where students use dry erase markers to write and paper towels to erase.

Cards are randomly drawn (shuffle, fan out and choose a card) to give parameters for the quadratic and students sketch the result. They then compare their sketch to others' work and discuss their thinking.

To introduce, model with the class how the cards work, then allow them to explore in their groups. Remind students that there will be situations when there is one solution, multiple solutions or no solution.

Circulate through the groups to help students and offer formative assessments. When teachable moments occur, stop the class and have a quick conversation before moving back to group work. Possible topics to discuss could be why we use vertex and standard forms of equations: pros, cons, preferences. Or if a graph isn't possible with the drawn parameters, what parameter could we change to make it possible.

Assessment and Reflection

The student worksheet can be used at any point for a quick summative assessment. The sheet contains areas for students to graph, write equations and also to reflect on their learning for one or two sets of parameters. Teachers can choose to give parameters or have students draw the parameters from card sets. The worksheet can be completed by students individually or with a partner.

The Cards

There are templates (*Quadratics_DealwithU_Cards* in the Resources download) for printing each set of cards, ideally on card stock. The templates can also be used as a guide for copying the information onto recipe cards.

There are four sets of cards (details below), getting progressively more difficult. Each set has an instruction card and some parameter cards.

For Sets B, C and D there are also challenge cards. These are optional challenges for students (or teachers) to provide an extension to the main task.

In addition to the Set cards, there are Number Cards (0 – ±10 that are used to find random values for parameters for Sets B, C and D. Ideally, Number Cards will be printed on different coloured card stock so they're easy to distinguish from the set cards.

Teacher Cards
There is a set of cards for teachers to distribute to students as desired to raise or lower the level of difficulty. Ideally, these will be printed on different coloured card stock so they're easy to distinguish from Set and Number cards.

Teachers can also carry blank cards to modify to distribute to students to decrease or increase the level of difficulty, as well as to practice a specific skill. Or students could choose their own parameters, which would offer another way to differentiate this for various student needs.

Using Technology
Card Sets C and D could also be used with DESMOS or other graphing software.

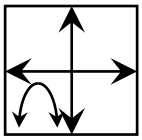
Card Sets

Set A: Qualitative Representation

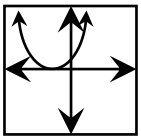
- Nine cards, kept in three separate piles - open, vertex, and crossing
- Draw one, two, or three cards
- Represent the result qualitatively (no numbers)

Opening	**Vertex**	**Crossing**
Opens up	Vertex in QI	Crosses the X-axis twice
Opens down	Vertex in QII	Crosses the X-axis once
	Vertex in QIII	Does not cross the X-axis
	Vertex in QIV	

Example 1: Draw: Opens down, Does not cross the X-axis, Vertex in QIII



Example 2: Draw: Opens up, Crosses the X-axis once



Example 3: Draw: Opens up, Crosses the X-axis twice, Vertex in QI
Not possible

Note:
Students may combine all the piles and draw two or three cards, which could lend itself to a discussion of mutually exclusive parameters.

Set B: Quantitative Representation

- Vertex form graphs
- Three sets of cards: a value, Number Cards and Challenge Cards
- Draw two Number Cards to determine vertex and one *a* value card
- Represent the criteria quantitatively
- Optional: Complete a Challenge

Example 1: Draw: $h = -7$, $k = -3$, $a = -1$

Result: U shaped graph drawn down through at V(-7,-3)

Example 2: Draw: $h = 2$, $k = 6$, $a = -2$

Result: U shaped graph drawn down through at V(2,6)

Students could write the vertex form as $y = -2(x-2)_2 + 6$ as a challenge

***a* value**	**Challenge**
$a = 1$	Represent the criteria in vertex form
$a = -1$	Represent the criteria in standard form
$a = 2$	Find the *y*-intercept
$a = -2$	If y is a perfect square, find the *x*-intercepts. If y is not a perfect square, why is it more difficult to find the *x*-intercepts?

Set C: More Quantitative Representation

- Two sets of cards: Number cards* and Challenge cards
- Draw three Number cards to determine the two *x*-intercepts and the *y*-intercept
- Represent the result quantitatively
- Optional: Complete a Challenge

Example 1: Draw: x_1-intercept = 2, x_2-intercept = -3 , *y*-intercept = -6
Result: U shaped graph drawn up through (2, 0) (-3, 0)

Example 2: Draw: x_1-intercept = -1 , x_2-intercept = 4 , *y*-intercept = 8
Result: U shaped graph drawn down through (-1, 0) (4, 0)
Students could calculate $a = -1/2$ as a challenge

Use with the Number Cards			Challenge
x_1-intercept is ___	x_2-intercept is ___	y-intercept is ___	• Represent the criteria in factored form • Solve for a • Find the vertex • Represent the criteria in standard form

* Teachers may want to separate Number Cards into all even or all odd piles so students will always be dealing with integers for vertex points.

Set D: Graphical representation created from solving algebraically or graphically

- Two sets of cards: Number cards and Challenge cards
- Draw three Number cards to determine a point on the graph (x, y) (not the vertex) and a or
  an x-intercept or y-intercept (student's choice)
- Graph the result solving algebraically (may be quite challenging!)
- Optional: Complete a Challenge

Example 1: Draw: $x = 2$, $y = -6$, y-intercept = -1

- Result: U shaped graph drawn up (or down) through the point (2,-6) and the y-axis at -1
- Algebraically students could solve in standard form $-6 = a(2)_2+b(2)+(-1)$, students would then have the option to pick a or b and solve for the other variable to make the equation true.

Example 2: Draw: $x = -3$, $y = 2$, x-intercept = 5

- Result: U shaped graph drawn up (or down) through the point (-3,2) and the x-axis at 5 and any other point on the x-axis.
- Algebraically students could solve in factored form $2 = a(-3-5)(x+2)$, students would then have the option to pick a second x-intercept and then solve for a to make the equation true.

Example 3: Draw: $x = -4$, $y = 5$, $a = -2$

- Result: U shaped graph drawn down through the point (-4,5) and at steps -2,-6, -10…
- Algebraically students could solve in standard form $5 = -2(-4)_2+b(-4)+c$, students would then have the option to pick b or c and solve for the other variable to make the equation true.

Use with the Number cards			Challenge
Goes through (*x, y*) (not vertex) *x* = ___ *y* = ___	x_1-intercept is ___ or *y*-intercept is ___	*a* = ___	• Complete the square from factored or standard form • Find the vertex • Find both x-intercepts • Take away one or add one parameter. Can you draw a graph with this information? Why or why not?

Teacher Cards

Modifiers for teachers to raise or lower the level of difficulty		
Does	1/2	-1/2
Does not	1/3	-1/3
<	1/4	-1/4
>	1/5	-1/5
x	*y*	*a*

Graphing Aerobics

Students use their bodies to make parabolas, based on given equations. This activity gets students moving which helps with focus, engagement, and retention of material. Try to incorporate movement into your math lessons wherever possible.

The activity starts with the teacher modelling a basic parabola (arms up and slightly curved), the vertex is at the base of neck.

Remind students of the order to move, SRT:

Stretch (compression or expansion)
Reflect (opening up or down)
Translate (move up or down or to right or left)

Equations can be displayed on large paper cards or projected Power Point slides. A sample set is provided (*Quadratics_GraphingAerobics.ppt x* in the Resources download). Students are shown the equation, given a few seconds to think about it and then together as a large group demonstrates it with their bodies. The teacher can act as a mirror, if you want students to move right, you move left.

The activity can happen over several days.

Day 1
- Step 1: move up and down "*q*"
- Step 2: move left and right "*p*"
- Step 3: combine 1 and 2

Day 2
- Step 4: reflect
- Step 5: combine 1 – 4

Day 3
- Step 6: expansion a bigger than 1 (use integers)
- Step 7: combine 1 – 6
- Step 8: compression $0 < a < 1$, use ½, ¼
- Step 9: combine 1 – 8

Then you can do Step 9 a bit every day (great hook), and as a review the day before the test and also right before the test to get their brains activated. It can also be a great review before a final exam.

Options: Teachers can play with or without music, in or out of their classrooms (taking students outside is ideal).

Online Games

Students can deepen their understanding through online quizzes and games. Both that were modelled in this activity had students working in groups.

Quizlet

Quizlet is a free online game and flashcard application. A custom quiz, Pre-Calc 11 Graphing Quadratic Functions is available at https://quizlet.com/_69m0ka

For this activity small groups of students, each with their own device, work as a team to solve questions on quadratics**.** Students are randomly grouped and need to sit with each other so everyone can see each other's screens. Students discuss the correct answer before choosing. A wrong answer will result in the team going back to the beginning. Paper may also be handed out so students can work out answers before responding.

Desmos

Desmos is a free online graphing application, including a set of eight games related to quadratics: https://teacher.desmos.com/quadratic

Marbleslides: Parabolas was the game used in this activity:
https://teacher.desmos.com/activitybuilder/custom/566b31784e38e1e21a10aade

The activity has students adjusting variables on parabolas to catch stars with marbles that are launched to slide off the parabolas. Students work in small groups of two or three on one device, a large screen is best. They progress at their own pace, working up through increasingly difficult situations.

Student Self-assessment for Understanding and Solving

Topic:_____

Name: :_____ Date:_____

Check all that apply to you at this time and fill in the blanks.

- Even with help, I do not understand.
- I am starting to get it.
- I can do this with help.
- I can do this with an example in front of me.
- I can do this on my own without help.
- I can explain to others how to do this.
- I have gained a deeper understanding of this topic.
- _____ (choose in-class activity) helped me to understand.
- In math, it helps me to _____ when I do not understand.
- It helps me to visualize what I learn.
- It helps me to draw pictures of what I learn.
- It helps me to move when I am learning something.
- It helps me to talk about what I am learning with other students.
- It helps me to have quiet time to think when I am learning.
- If I could do this unit over again, I would _____.

  * For teachers: This checklist could be edited to provide more opportunities for writing and more open-ended questions.