Predicting a Catastrophe

Foundations of Mathematics and Pre-calculus 10

Activity Plan	Design Thinking
This is the outline of our activity plan, with all the necessary links and materials. Description of Learning Activity: This activity is an investigation into a potential food shortage. Population and food production data will be analyzed, modelled (both by hand and using technology) and compared to predict the timing of a potential food shortage. Videos on this activity's design considerations, classroom delivery and teacher/student reflections are available on Focusing on Competencies in Math.	*Here is where the writers have outlined the thinking, conversations and decision making while developing the plan.*

Big Ideas:
• Representing and analyzing situations allows us to notice and wonder about relationships.
• Constant rate of change is an essential attribute of linear relations and has meaning in different representations and contexts.
 	
Core Competencies:
• Communication
  ○ Communicating: connecting and engaging with others (to share and develop ideas)
  ○ Collaborating: working collectively
• Thinking (Critical and Reflective)
  ○ Analyzing and critiquing
  ○ Questioning and investigating
  ○ Designing and developing
• Personal and Social
  ○ Personal Awareness and Responsibility: self-advocating
  ○ Social Awareness: contributing to community and caring for the environment
 	
Curricular Competencies:
• Reasoning and Modelling
  ○ Explore, analyze, and apply mathematical ideas using reason, technology, and other tools
  ▪ examine the structure of and connections between mathematical ideas
  ▪ predictions, generalizations, conclusions drawn from experiences (e.g., with puzzles, games, and coding)
  ○ Model with mathematics in situational contexts
  ▪ use mathematical concepts and tools to solve problems and make a decision
 	
• Incorporate by:
  ○ Exploring the First People Principles of Learning
  ▪ Learning involves recognizing the consequences of one's actions
  ○ Exploring cultural practices and knowledge of First Peoples and identifying mathematical connections
	All of the Core Competencies are addressed in this activity plan, however with our focus being on the Curricular Competency of Reasoning and Modelling that will be the only work formally assessed.
	
	This activity has been designed as an introduction to systems of equations for students in Foundations of Mathematics and Pre-calculus. Students would have previously been exposed to linear functions. While this activity could encompass many of the Curricular Competencies we have chosen to focus on Reasoning and Modelling.
	
	While designing the activity and assessment, we struggled with staying true to this focus. We had to remind ourselves that even though many other competencies could be demonstrated through this activity, our focus was on Reasoning and Modelling.

Content:
• Foundations of Mathematics and Pre-Calculus 10
  ○ Functions and Relations: connecting graphs and context
  ○ Functions and Relations: domain and range in situational contexts
  ○ Linear functions: connections between representations: graphs, tables, equations
  ○ Systems: solving graphically
  ○ Systems: solving problems in situational contexts
 	
Other Possible Content Areas:
• Foundations of Math 12
  ○ Representations: using characteristics of a graph to identify these functions
  ○ Regression analysis: polynomial and exponential
  ○ Regression analysis: applying the appropriate regression model
• Pre-Calculus 12
  ○ Exponential: graphing
  ○ Exponential: solving problems in situational contexts
  ○ Polynomial: solving equations graphically
• Workplace 11
  ○ interpreting graphs**:** investigating graphs in the media (e.g., news articles, blogs, social media, websites, advertisements)
  ○ interpreting graphs: how data and media influence social justice issues and personal decisions
 	
Cross-curricular links to Social Studies:
• Social Studies 10
  Curricular Competencies
  ○ Use Social Studies inquiry processes and skills to ask questions; interpret and analyze ideas; and communicate findings and decisions
  ○ Assess how underlying conditions and the actions of individuals or groups influence events, decisions or developments, and analyze multiple consequences
• Human Geography 12
  Content
  ○ Demographic patterns of growth, decline, and movement
  ○ Global agricultural practices
  ○ Relationships between natural resources and patterns of populations settlement and economic development	

Check out the data below; it shows the population of a country and how many people it can feed based on their agricultural production. ***Note for teachers:*** *The population data is for Sierra Leone. Given the fact that countries rely on imports and exports for food and agriculture, we could not find food production data for a country that did not also either import or export food. As such, the data below is fictitious.*	*We were looking through the NCTM magazine Mathematics Teacher (October 2018) and found an article called Teaching Human Rights through Mathematics, by Blair Izard.* Izard, B. (2018, October). "Teaching Human Rights Through Mathematics." *Mathematics Teacher,* 112(2), p. 114 – 119.

Year	Population (number of people)	Food Production (number of people)
1975	2,993,876	6,400,000
1980	3,365,441	7,000,000
1985	3,799,550	7,100,000
1990	4,312,246	8,100,000
1995	4,274,819	9,400,000
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In a group of two or three, use the grid provided to graph the data; you will want to make some adjustments to the scale.

What do you notice from the data? What do you wonder?

Model the relationship with two functions.
student handout is provided at the end of this document

We were excited by the topic of a potential global food shortage and an opportunity to link math not only to the humanities but also an opportunity to bring in an Indigenous perspective into the activity.

When thinking about this activity, we wanted to start with something to engage students and spark curiosity and questioning.

We were thinking the easiest way to graph this data would be on a large piece of chart paper, or an 11x14 grid so that students had more room to see the full relationship.

A Learning Sequence

Activity
Introduction As a class discuss graphs from https://www.nytimes.com/column/whats-going-on-in-this-graph What do you wonder? What do you notice?
Choose one of the articles below and have students read it over. **Possible articles:**

After doing this activity several times, we think it would work best split over two classes, taking approximately two hours to complete. We would suggest the first part to be the introduction, graphing by hand, and working through the student hand out. The second day would be the Desmos activity and self-reflection.

https://www.cbc.ca/news/indigenous/tsou-ke-nation-green-energy-leaders-1.4067833 **https://www.cleanenergybc.org/news/first-nations-powering-up-b-c** **https://www.cbc.ca/news/politics/first-nations-renewable-energy-projects-1.4348595** Discuss how these First People's communities are working towards a clean future and the importance of taking care of our environment for future generations. Use this as a segue to introduce the video and the idea of overpopulation.	*We were excited about the opportunity to bring First People's perspective authentically into the math classroom, as we find it is often difficult to make a meaningful connection. When looking at the data, ask questions such as "What would cause a drop in food production" probing students to think about environmental factors that affect the growth of food.*
Show video and intro topic: https://www.youtube.com/watch?v=VcSX4ytEfcE -show full video (2:33)	
Analyzing the Data Handout student paper. Students discuss data with a partner. The teacher asks; what did you notice? What do you wonder? Share observations If necessary, prompt students towards the idea of a potential food shortage.	
Representing the Data In groups of two-three, students discuss how they would represent the data. What do you notice? What do you wonder? Share observations. How did you analyze the data? Did you consider multiple ways to represent the data? Share your thinking.	*Formative Assessment: Can students graph a linear function from a table of values?*
Graphing the Data In groups of two-three, students graph data on a provided grid (could be on paper or a vertical surface). When done, try to create a function which models the data and predict when the lines will intersect What do you notice? What do you wonder? Share observations. Can you write a function for the relationships? Share your thinking.	*We are assuming students will begin by graphing two linear equations. This is a good starting point and discussion piece, but we want to let them sit with the two linear equations and allow them to try to create functions and solve the problem before recognizing that there is a problem with the model.*
Reflecting on the Graph Share models and graphs posted around the room. Share predictions: Will they intersect? When will they happen? Is this a logical answer? Teacher explanation: Sometimes, data doesn't fit a linear model very well. We are hoping that based on the video watched, students will realize that the intersection should happen earlier than what their predictions will be using two	

linear models. At this point, we would introduce students to the idea of a different type of function being needed to get a more reasonable answer. In this case, an exponential function might be a better representation; we're going to try using Desmos to model our data.	
Using Technology Students log in to laptop and access Desmos activity (see below for link). The teacher restricts students to screens 1 and 2.	
Snapshot 1-3 student answers: how the graphs look similar or different and discuss. Then, allow students to work on screens 3 and 4.	
Snapshot 1-3 student answers: differences in the exponential model. Allow students to complete the activity.	
Students complete the Desmos activity.	
Self-Assessment Discussion and self-assessment (see below for self-assessment rubric)	

Desmos Activity

https://teacher.desmos.com/activitybuilder/custom/5c82d0707b11da0c1e3a1c1e

Screen	Topic
1	Show graphed data Ask: Does this look like your graph? Why or why not?
2	Students predict based on their models
3	Students use sliders to create a line of best fit for the food production data What do *m* and *b* indicate?
4	Students use sliders to create an exponential function fitting population data Why is an exponential model better than a linear one in this case?
5	Students examine the two functions together
6	Show point of intersection. What does this mean? How close were your predictions?
7	Students examine the equation of the linear function and determine the meaning of *m* and *b*
8	Highlight a couple of outlying points. What could have happened here? How could climate change affect future food production?
9	Sustainability (First Peoples) question #1

Desmos is a great tool to use because the teacher can control the pace students move through the activity, as well as pause the activity to discuss ideas as a class and can view what all students are doing at the same time.

Formative Assessment: This is a great time to see if students understand how slope and y-intercept affect a linear function.

Formative Assessment: Do students understand the solution to a system of equations?

Formative Assessment: This is an opportunity to see if students can put context to a linear function.

10	Sustainability (First Peoples) question #2	

Teacher Assessment Rubric - Reasoning and Modelling

	Emerging	Developing	Proficient	Extending
Curricular Competency: model with mathematics in situational contexts	Can create a model (equation or graph) to represent data	Can create equations and graphs to represent data	Can create equations and graphs to represent data and discuss the benefits of both representations	Can create equations and graphs to represent data, select a preferred model and justify the decision
Curricular Competency: explore, analyze, and apply mathematical ideas using reason, technology, and other tools	Can recognize the short-comings of a linear model for the data	Can use Desmos to create regression models for the data	Can recognize and explain the meaning of the intersection of the two functions	Can fully explain the relation between the two functions, in context
Content: functions and relations: connecting data, graphs, and situations	Can recognize there are different types of relationships	Can recognize that one relation is linear and one is non-linear	Can describe the rate of change for both linear and nonlinear function	Can describe the rate of change for both linear and nonlinear function, able to recognize that the intersection point is the solution

Task-specific Evidence - These points are teacher selected evidence that shows the students' proficiency in reasoning and modeling based on this specific activity. Teachers can modify these tasks specific evidence for different activities while keeping the general description of the curricular competency.

Student Handout

This would be given to students at the beginning of the activity to work through the first part of the modelling. Students would then transfer to Desmos to continue the rest of the activity.

Version 1 offers less guidance for students who are used to working through activities like this.

Version 2 embeds more specific questions to guide students' thinking.

As we were developing the student handout, we realized that, depending on previous classroom experiences, some students may struggle with the open-ended questions like 'What do you notice?' If they have not had experience with these types of questions previously, we felt more guiding questions could be helpful for both the

	students and the teachers.
Group Members: _____ Predicting A Catastrophe (version 1) Check out the data below; it shows the population of a country and how many people it can feed based on their agricultural production for each year. Take some time to look at the data and graph it on the grid provided. What do you notice? What do you wonder? Can you show this relationship in another way?	*If students need some guidance you can ask them questions about what type of relationship they are looking at, such as: How is the data changing from one point to the next? Do you see a pattern? Are the population and food production changing at the same rate? Does one change more quickly? Can you relate this to what we have been learning?* *You could also ask about the scale they are choosing to use to graph the data. Encourage them to set 1975 to be year 0 and go up from there to have a more manageable scale.* *We want students to get to wonder when, if ever, food will run out if growth continues on the same track. If students want to answer this question, ask them how they would find out? They should conclude that the lines will have to be extended to see if they meet.*

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We want students to try to write the relationship as an equation - if they are thinking they are both linear, they should be able to write two functions, if they have identified that one is not linear, just ask for one equation.

Do you think the model you have created accurately represents the data? Why or why not?

We want students to recognize that the model is not completely accurate (either from looking at the line of best fit and seeing that it doesn't fit the data that well or from extending it and not thinking they have a reasonable answer) and that it could probably be improved upon, at this point, however, they may not know how.

Student Self-Assessment

Curricular Competency Reasoning and Modelling	**Task Specific Evidence**
I analyzed the data to look for a pattern	☐ I considered how the data changed with respect to time. ☐ I noticed that one relation increased faster. ☐ I was able to use the term 'rate of change' when describing the data.
I considered multiple ways to show the data	☐ I considered describing the data in words, an equation and/or a graph. ☐ I was flexible in my thinking and listened to my group members' ideas.
I created multiple models to show the data	☐ I have a function, equation and/or graph that represents the data. ☐ I have a model that is scaled to fit the data. ☐ I considered the domain to reflect this real-world scenario.
I made a prediction based on my model	☐ I have a model that was scaled appropriately to make a prediction. ☐ I recognized the intersection of the functions was a solution to the problem. ☐ I was able to describe the limitations of my model.

This is the point where we would introduce them to the idea of the population growth being an exponential function and move on to the Desmos activity.

The first part of the self-assessment could be done at the end of the student handout, however, the reflection piece should be saved until the Desmos activity has been completed.

Core Competency Reflection:

1. What parts of this activity did you find interesting? Surprising?

2. What did you find went well in this activity?

3. What did you find challenging?

4. Was it helpful to work in a group? Why or why not.

5. What would you do differently in a future activity to improve?

We want students to recognize that through this activity, even though the assessment focus was on Reasoning and Modelling, they were engaging in all of the Core Competencies while participating in the activity.

Group Members: _____

Predicting A Catastrophe (version 2)

The second version of the student handout has more guiding questions to assist students who have not done an investigation like this before.

Part 1: Analyzing the Data

Check out the data below. It shows the population of a country and how many people it can feed based on their agricultural production for each year.

Year	**Population** (number of people)	**Food Production** (number of people)
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Take some time to look at the data and discuss your observations with your group members. Write down your observations; be sure to add detail and use vocabulary from previous chapters.

Part 2: Representing the Data

Discuss with your group how you could represent this data differently. Write down your ideas.

What are the advantages and disadvantages of your ideas?

Part 3: Graphing the Data

As a group, create a graph of the data. Make sure to decide on an appropriate scale before you make your graph. What assumptions are you making as you graph? List three.

Part 4: Reflecting on the Graph

After graphing the data, what did you notice?

Can you identify the type of relationship(s) present?

What do you wonder? When do you think this country might run out of food?

Do you think the model you have created accurately represents the situation? Explain.

Self-Assessment

	Curricular Competency	**Task-specific Evidence**
Part 1	I analyzed the data to look for a pattern.	☐ I considered how the data changed with respect to time. ☐ I noticed that one relation increased faster. ☐ I was able to use the terms: relation, rate of change, constant/non-constant, linear/non-linear and independent/dependent variables.
Part 2	I considered multiple ways to show the data.	☐ I considered describing the data in words, an equation and/or a graph. ☐ I was flexible in my thinking and listened to my group members' ideas. ☐ I considered assumptions I would be making.
Part 3	I created multiple models to show the data.	☐ I have a function, equation and/or graph that represents the data. ☐ I have a model that is scaled to fit the data. ☐ I considered the domain to reflect this real-world scenario.

Part 4	I made a prediction based on my model.	□ I have a model that was scaled appropriately to make a prediction. □ I recognized the intersection of the functions was a solution to the problem. □ I was able to describe the limitations of my model.	

Core Competency Reflection:

1. What parts of this activity did you find interesting? Surprising?
2. What did you find went well in this activity?
3. What did you find challenging?
4. Was it helpful to work in a group? Why or why not.
5. What would you do differently in a future activity to improve?

Teacher Self Reflection:

1. Was meaningful learning taking place? Were students engaged? Were they questioning?
2. What parts of the activity went well for your students?
3. What parts could be improved upon?
4. If you were to do it again, how would you change the lesson (either in structure or the delivery of it)?

After doing this lesson a number of times, I found that the students enjoyed the 'real-life' situation. It put into context the math we had been working on in class without a contrived scenario. I was surprised that almost all of the groups struggled with putting the time (years) onto the graph with an appropriate scale. Doing it over two classes, instead of one, allowed students to immerse themselves in the data, gave them more time for questioning, figuring out how to graph the data, and they reflected upon what the model meant. I was pleasantly surprised that some of my students were able to make the connection between this activity and linear systems, once I introduced the topic.