The Beauty of Math

Pre-calculus 12

Suggested time: 3 hours
over the duration of the course/unit (ongoing project)

Description/Rationale

The competency focus for this activity is Reflecting and Connecting. Students have ownership of their final product and get to show off their mathematical abilities through modeling pictures as transformations of functions, including domain and range.

Teachers can introduce this project on the very first day of class or at the start of the transformations unit. As the class goes through the term or unit, students are encouraged to connect the math they are learning in class to the project and reflect on it. Students will work on the project throughout the term or unit.

This project is a way to celebrate math. Students end up with a beautiful math project they can be proud of. In this activity, students are focused on connecting mathematics to a drawing of their choice and reflecting on their mathematical thinking as they find and refine functions to best represent their drawing. The project also helps students work on Core Competencies: communicating, critical and reflective thinking, and creative thinking.

The resulting images can be printed and proudly displayed so that the whole school has the opportunity to celebrate the beauty of math. This helps create a positive attitude towards math in your school. Math is beautiful and amazing and should be celebrated!

Videos on this activity's design considerations, classroom delivery and teacher/student reflections are available on Focusing on Competencies in Math.

Big Ideas

- Transformations of shapes extend to functions and relations in all of their representations.
- Understanding the characteristics of families of functions allows us to model and understand relationships and to build connections between classes of functions.

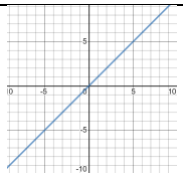
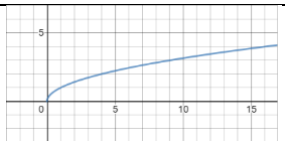
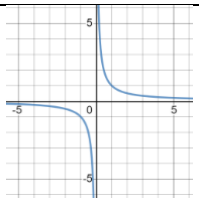
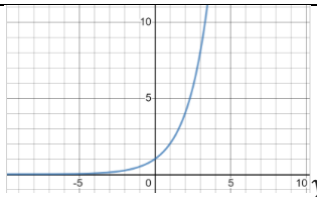
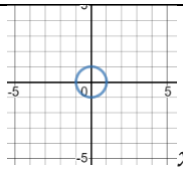
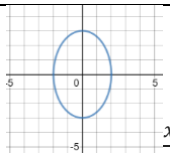
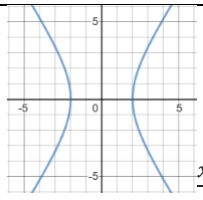
Curricular Competencies: Connecting and Reflecting

- Reflect on mathematical thinking
- Connect mathematical concepts with each other, other areas, and personal interests
- Use mistakes as opportunities to advance learning

Introduction to the Project

Before this project is started, students should know several relations and transformations, inverses, and domain and range. Transformations include vertical and horizontal translations, stretches, and reflections.

Some Common Parent Relations

Linear	$y = x$	
Radical	$y = \sqrt{x}$	
Reciprocal	$y = \frac{1}{x}$	
Exponential	$y = a^x$	 $y = 2^x$
Circle	$x^2 + y^2 = r^2$	 $x^2 + y^2 = 1$
Ellipse	$\frac{x^2}{h^2} + \frac{y^2}{k^2} = 1$	 $\frac{x^2}{4} + \frac{y^2}{9} = 1$
Hyperbola	$\frac{x^2}{h^2} - \frac{y^2}{k^2} = 1$	 $\frac{x^2}{4} - \frac{y^2}{9} = 1$

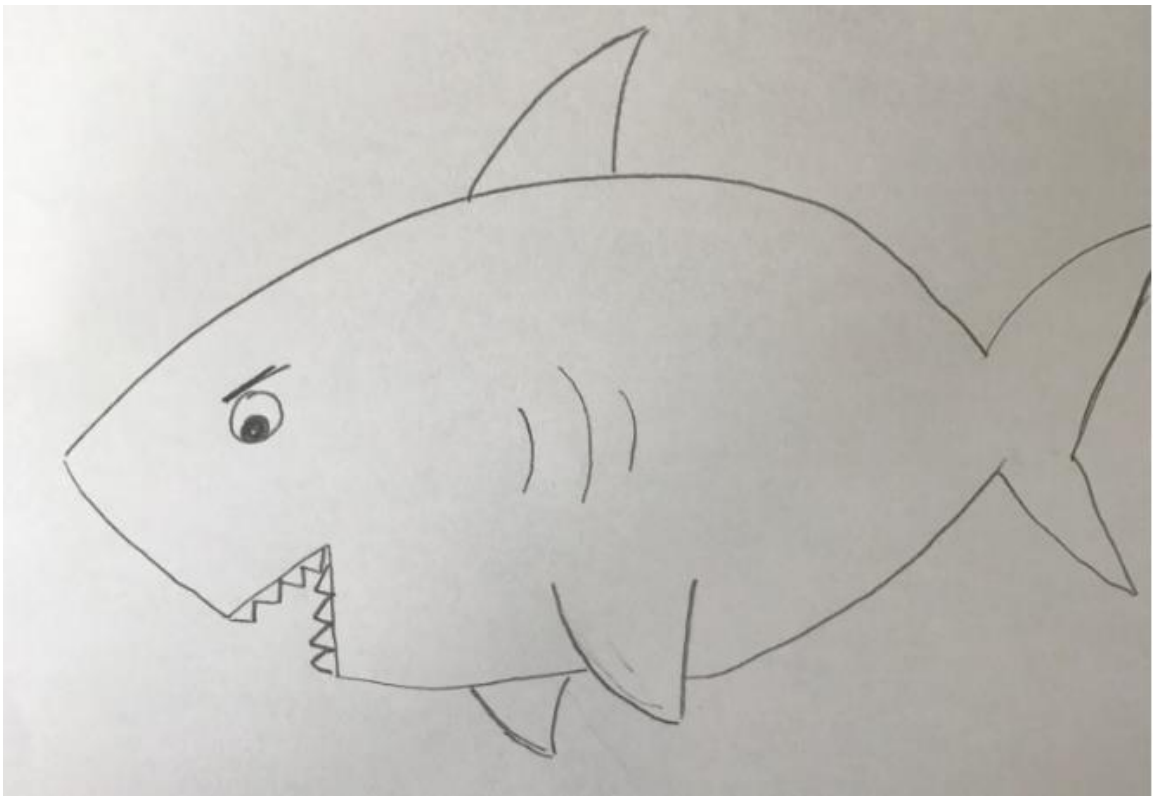
Desmos Classroom Activities

To help teach transformations and parent relations, Desmos Classroom Activities can be used. Here's an introductory video: Welcome to Teacher Desmos.

- In "Desmos Classroom Activities" teachers can use already made activities or they can custom make their own.
- Here is an example of a custom made Desmos Activity.
- Student can play with pictures and relations in Graphing Calculator Desmos. Students sign in to Graphing Desmos Calculator.
- This video explains About Desmos.

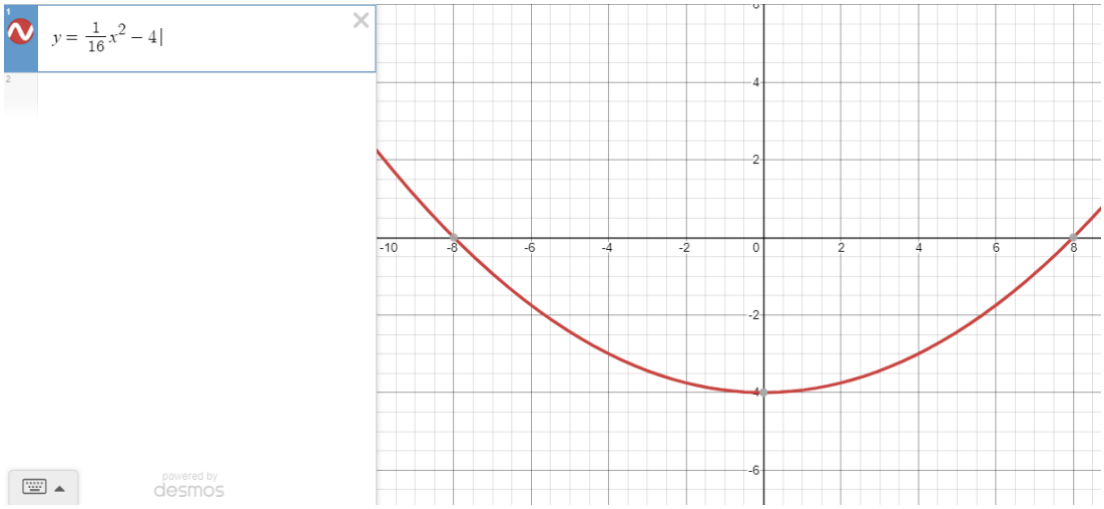
The Project

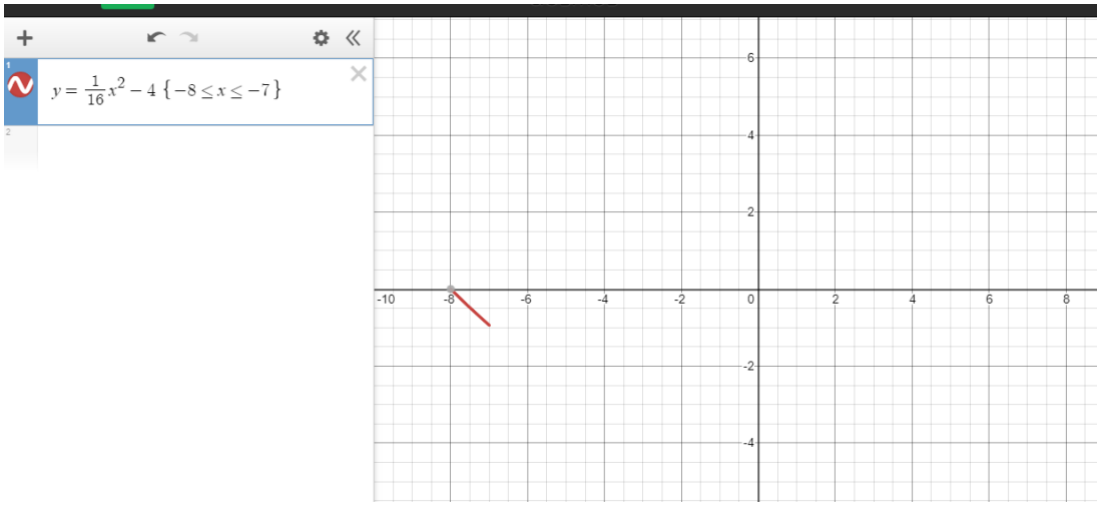
Students sketch or find a picture that could be drawn using the functions and relations they have learned. Learning is holistic, reflexive, reflective, experiential, and relational, so their sketch is just a starting point. Students are welcome and encouraged to adjust their sketch throughout the project.

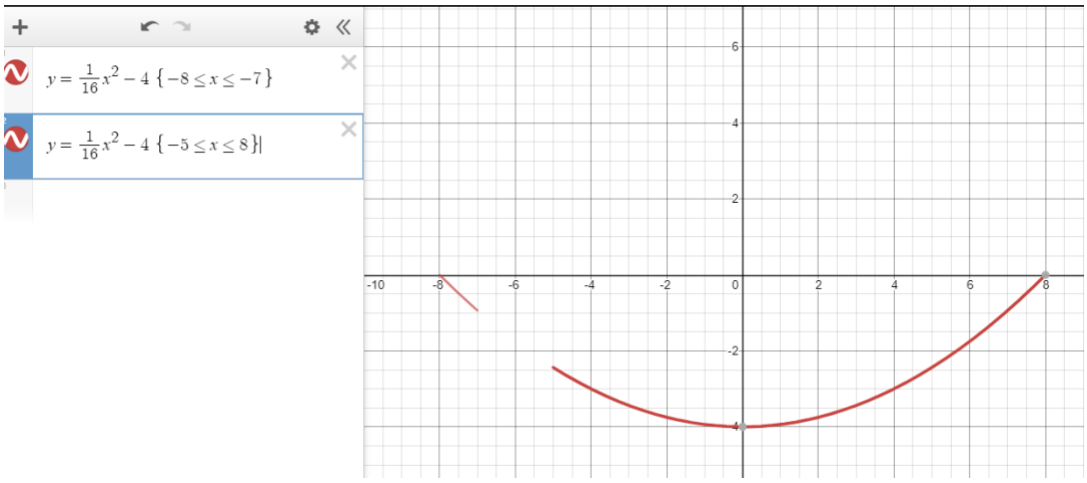


Students sign into Graphing Desmos Calculator (an online graphing calculator where students can save their work). Desmos graphs in real time, without waiting for a complete function. In other words, the graph changes as the function is entered so students can easily see how each parameter of an equation affects the graph. Desmos is AWESOME for 'playing around' with domain and range with very little consequence. By using technology (i.e. Desmos), students can make mistakes in a safe and easily correctable learning environment. Students also receive direct feedback and can see how the changes they make to the function equation affects the graph. The differences in functions and relations for horizontal and vertical translations or stretches will be apparent to students.

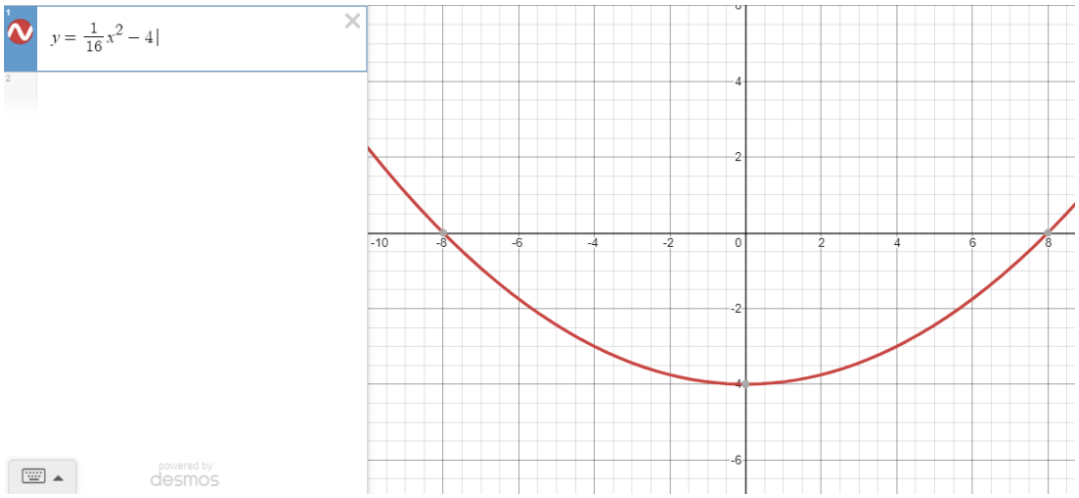
Here is an example of how students could play with relations to create a shark:

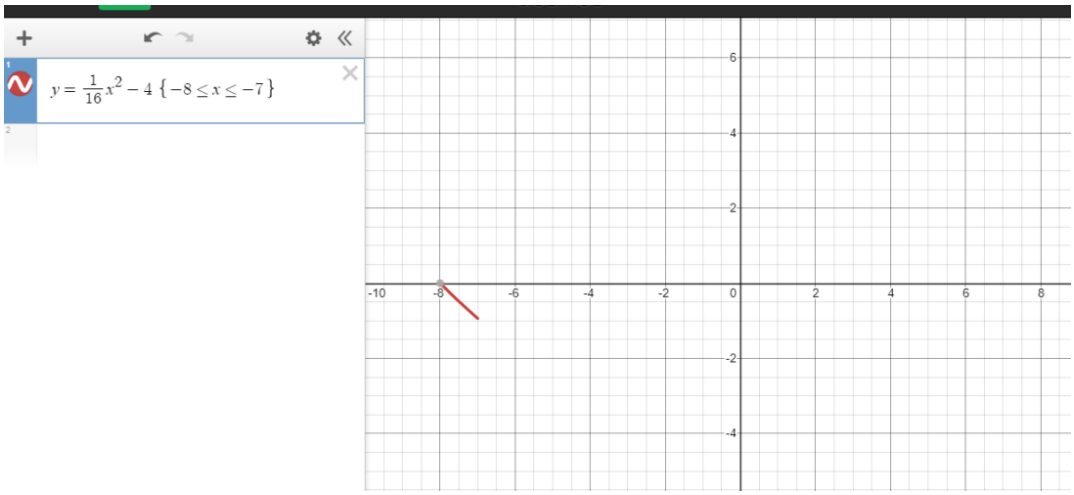


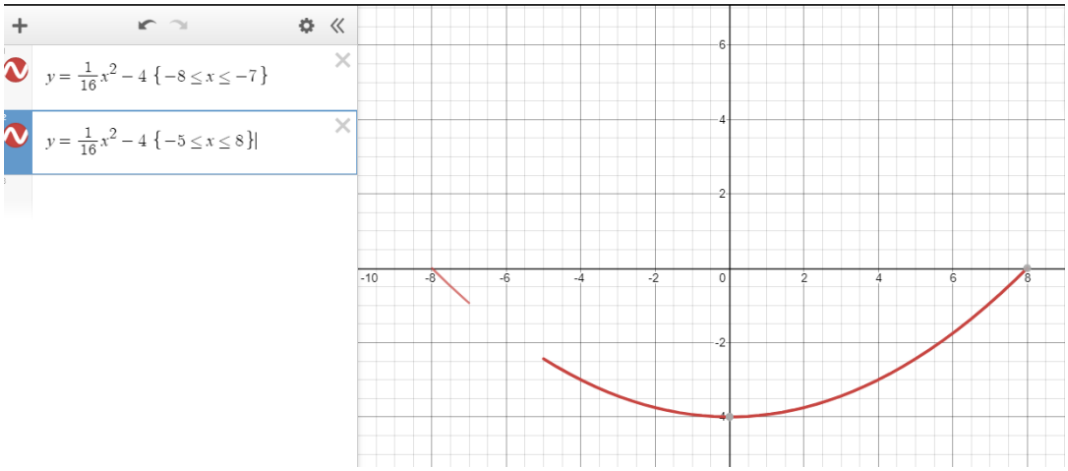




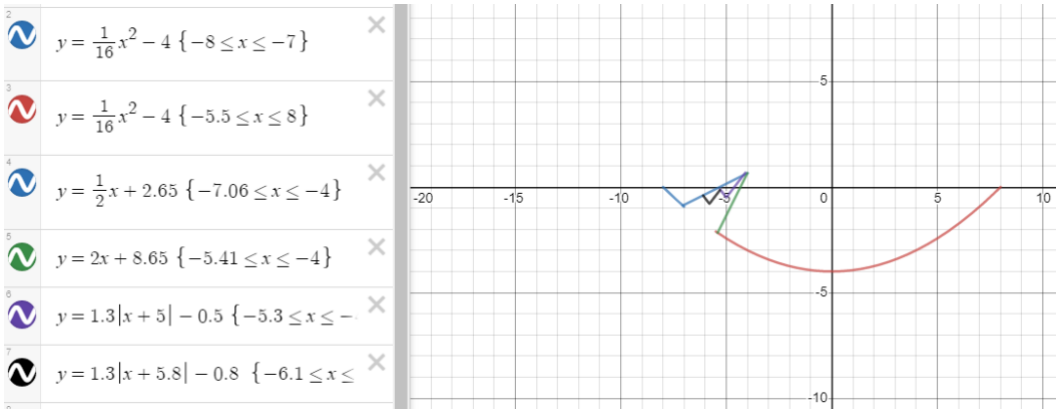
- Restricting the Domain of a parabola can be transformed into the bottom of a shark:





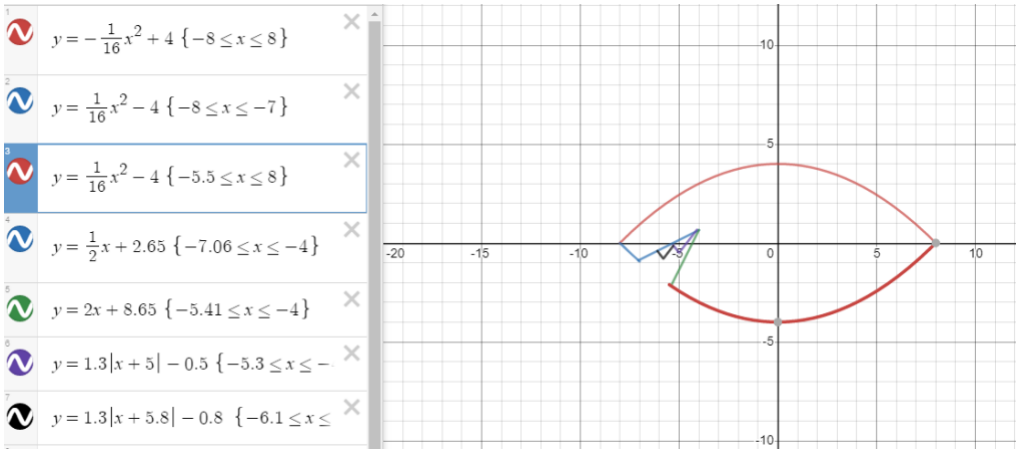


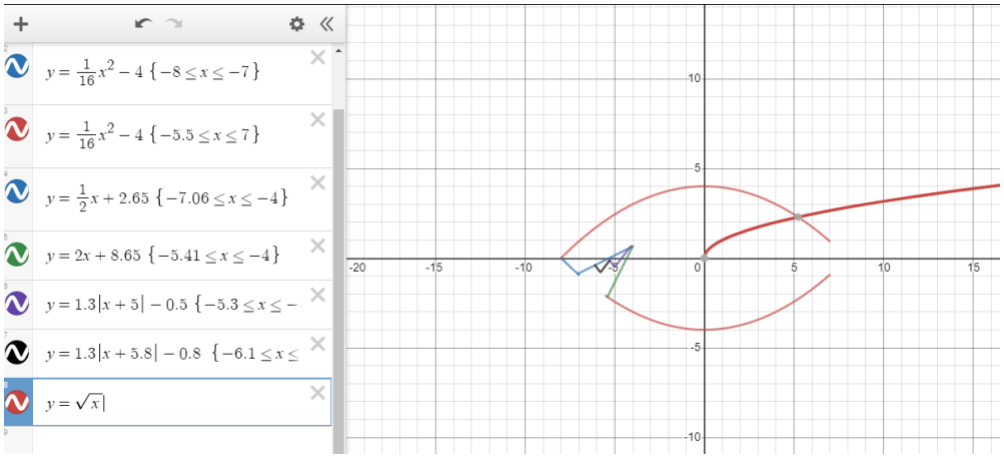
- Lines and absolute value functions are added to start creating a mouth.

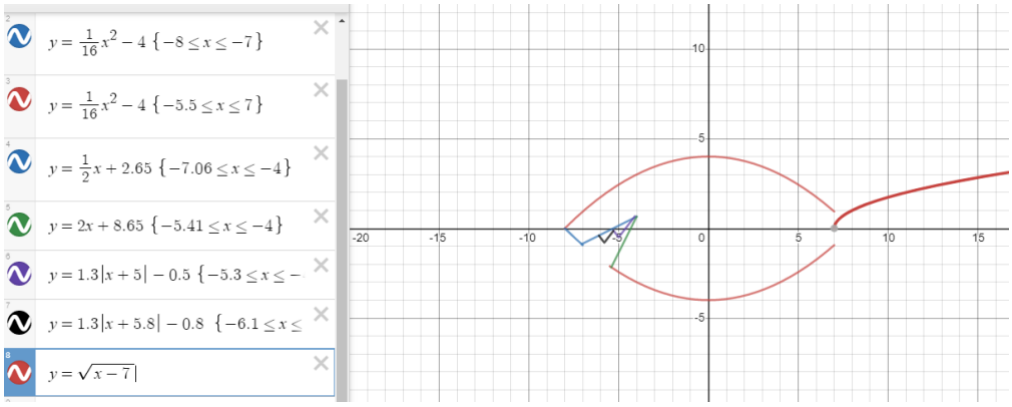


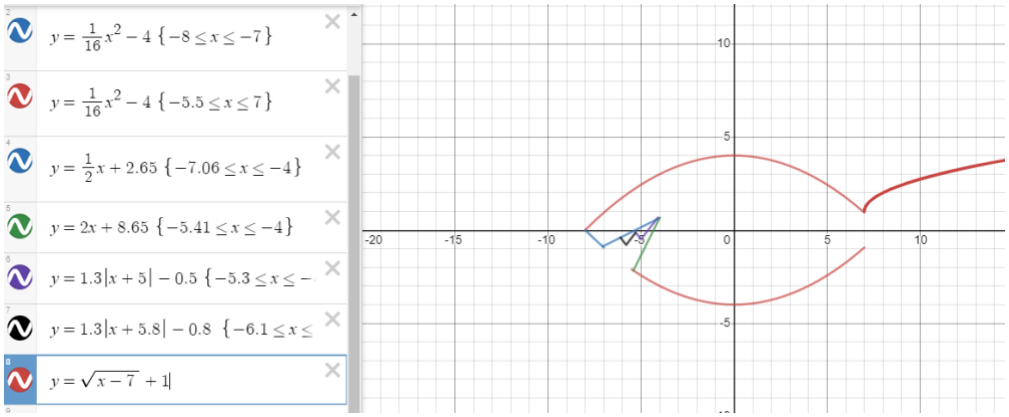
- Continuing to play with different functions and domain and range to see what works and what does not work. This is a great place to learn from mistakes. When students are doing this, they need time here to play—learning involves patience

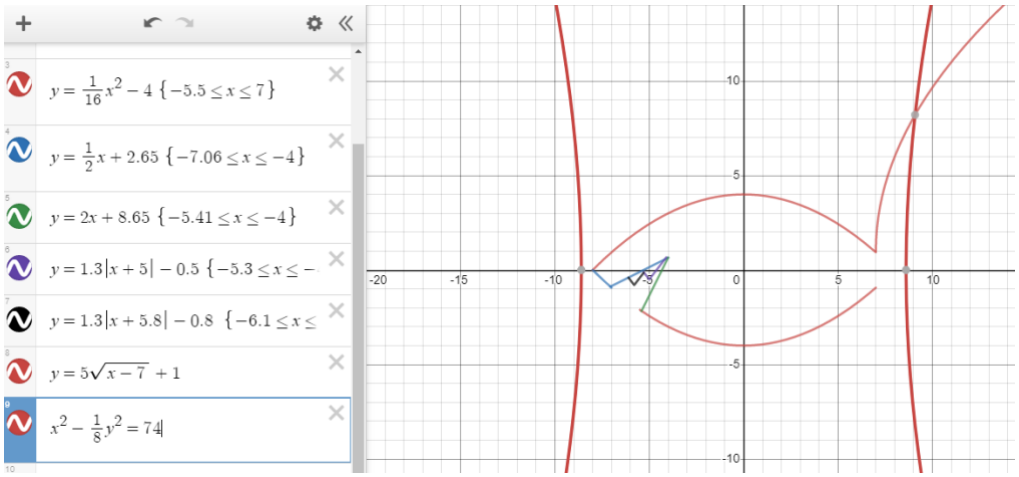
and time. This is also a great opportunity for students to talk to each other and reflect on what works.








$y = \frac{1}{16}x^2 - 4 \{-8 \le x \le -7\}$
$y = \frac{1}{16}x^2 - 4 \{-5.5 \le x \le 7\}$
$y = \frac{1}{2}x + 2.65 \{-7.06 \le x \le -4\}$
$y = 2x + 8.65 \{-5.41 \le x \le -4\}$
$y = 1.3|x + 5| - 0.5 \{-5.3 \le x \le -$
$y = 1.3|x + 5.8| - 0.8 \{-6.1 \le x \le$
$y = \sqrt{x - 7} + 1$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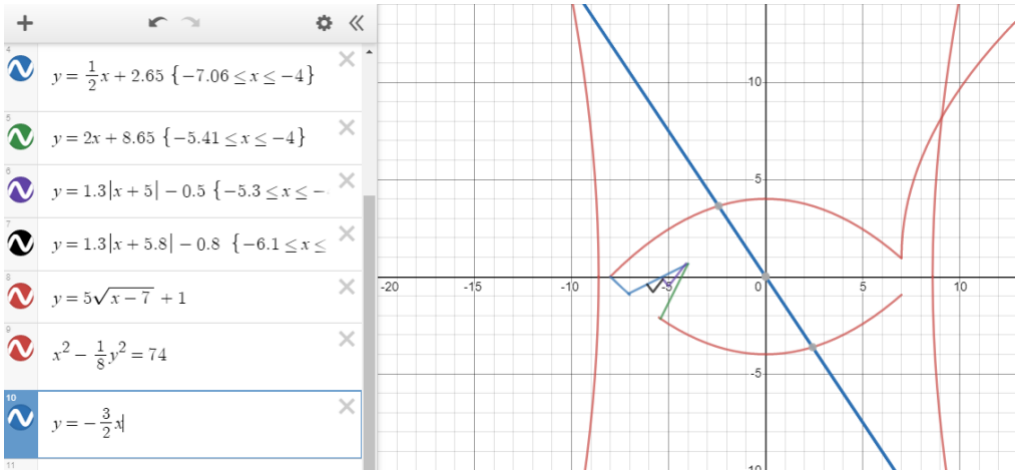
$y = 5\sqrt{x - 7} + 1$
$x^2 - \frac{1}{8}y^2 = 74$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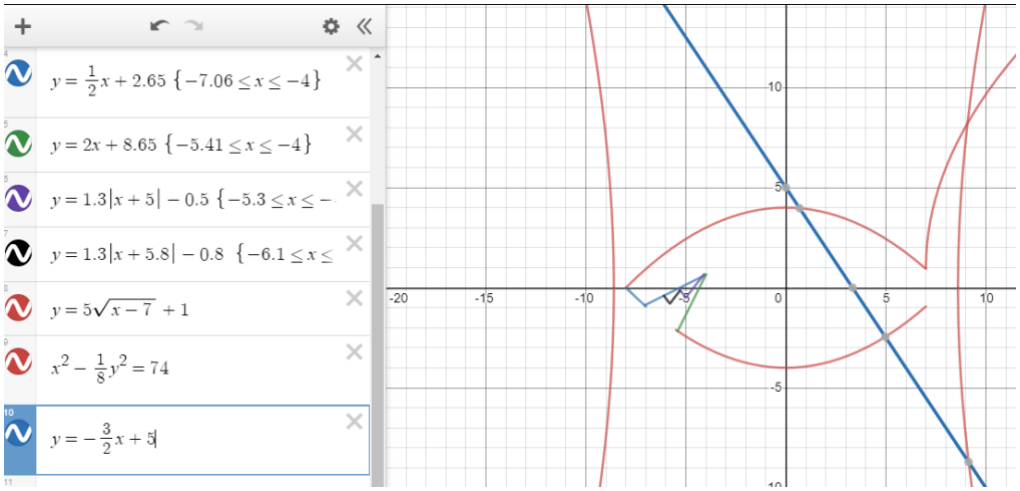
$x^2 - \frac{1}{8}y^2 = 74$
$y = -\frac{3}{2}x$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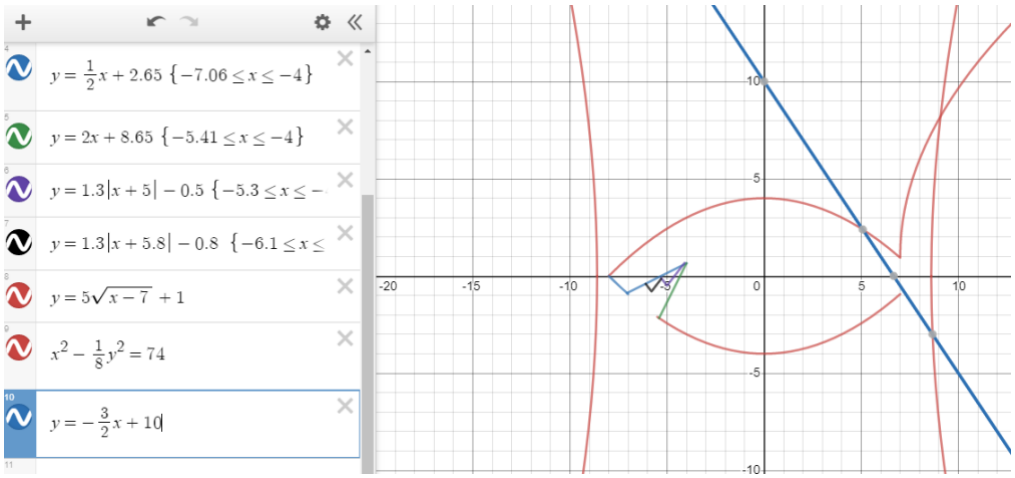
$y = 5\sqrt{x - 7} + 1$
$x^2 - \frac{1}{8}y^2 = 74$
$y = -\frac{3}{2}x + 5$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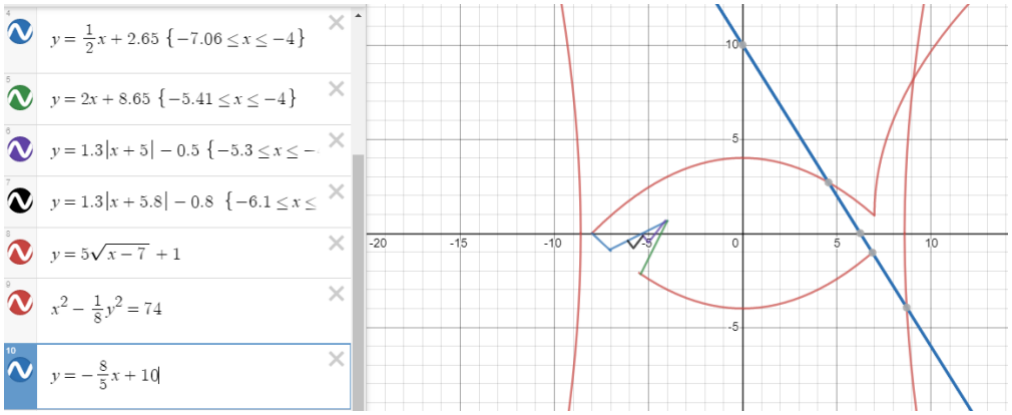
$y = 1.3|x + 5.8| - 0.8 \{-6.1 \le x \le$
$y = 5\sqrt{x - 7} + 1$
$x^2 - \frac{1}{8}y^2 = 74$
$y = -\frac{3}{2}x + 10$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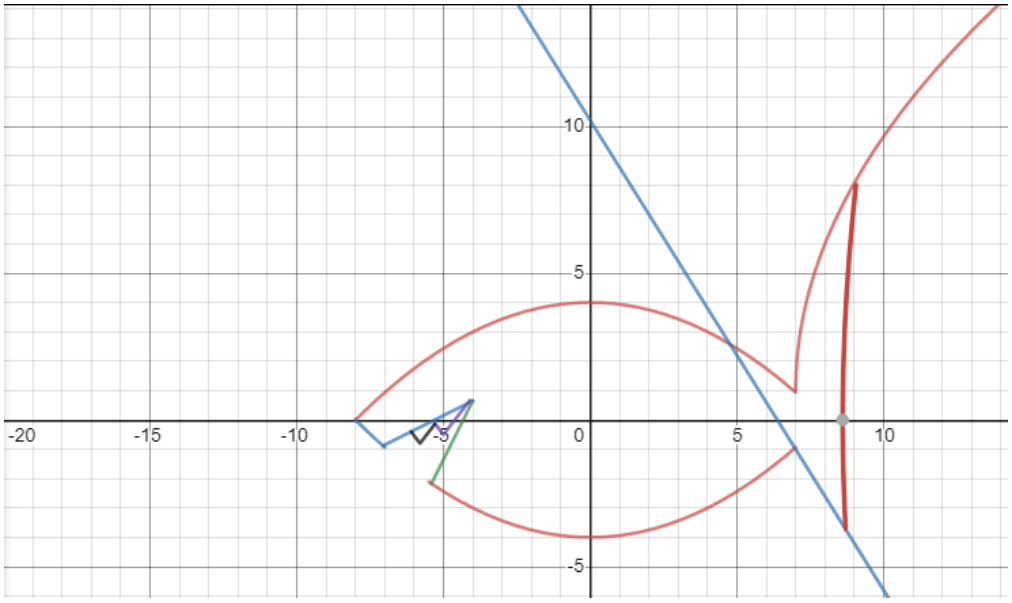
$y = -\frac{8}{5}x + 10$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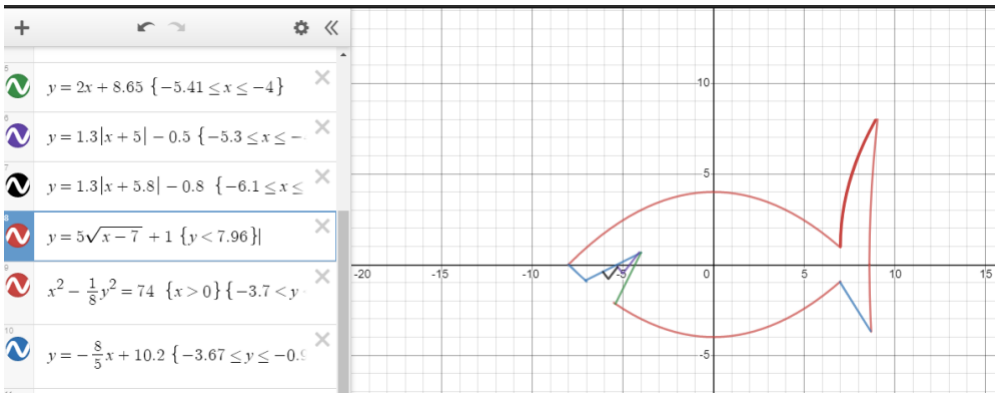
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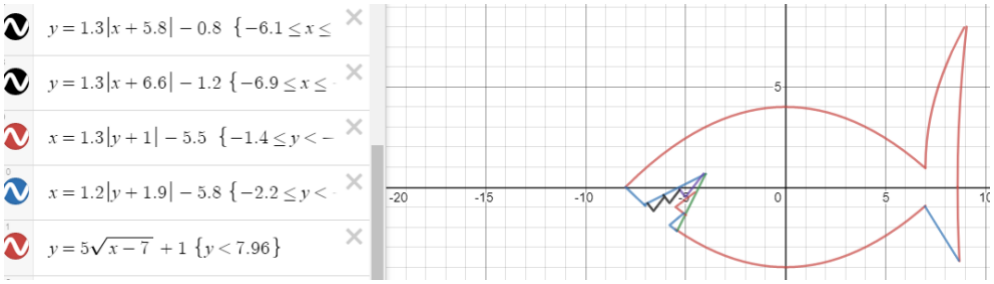
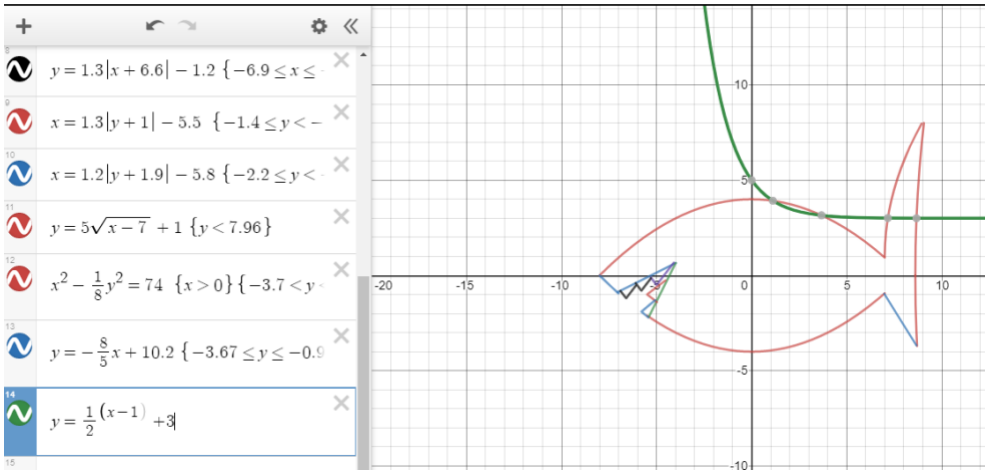
$y = 5\sqrt{x - 7} + 1 \ \{y < 7.96\}$
$x^2 - \frac{1}{8}y^2 = 74 \ \{x > 0\} \{-3.7 < y$
$y = -\frac{8}{5}x + 10.2 \ \{-3.67 \le y \le -0.$


$y = 1.3|x + 5.8| - 0.8 \ \{-6.1 \le x \le$
$y = 1.3|x + 6.6| - 1.2 \ \{-6.9 \le x \le$
$x = 1.3|y + 1| - 5.5 \ \{-1.4 \le y < -$
$x = 1.2|y + 1.9| - 5.8 \ \{-2.2 \le y <$
$y = 5\sqrt{x - 7} + 1 \ \{y < 7.96\}$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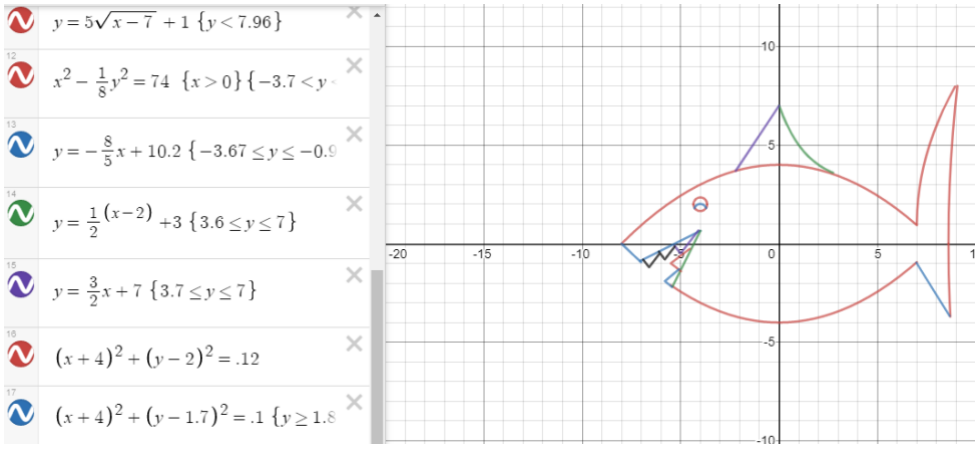
$x^2 - \frac{1}{8}y^2 = 74 \{x > 0\} \{-3.7 < y$
$y = -\frac{8}{5}x + 10.2 \{-3.67 \le y \le -0.9$
$y = \frac{1}{2}(x-1) + 3$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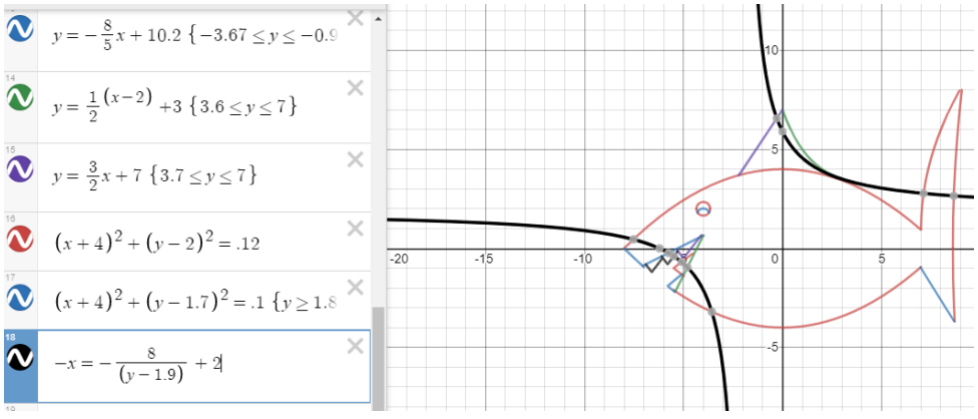
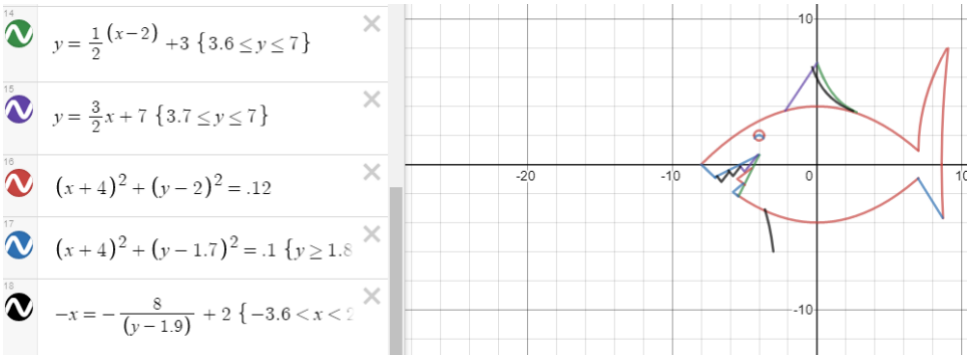
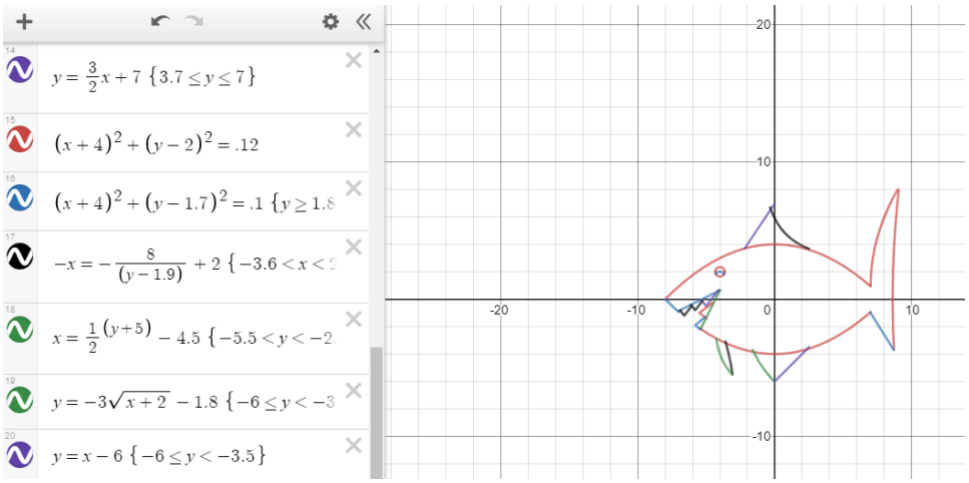
$x^2 - \frac{1}{8}y^2 = 74 \{x > 0\} \{-3.7 < y$
$y = -\frac{8}{5}x + 10.2 \{-3.67 \le y \le -0.9$
$y = \frac{1}{2}(x-2) + 3 \{3.6 \le y \le 7\}$
$y = \frac{3}{2}x + 7 \{3.7 \le y \le 7\}$
$(x+4)^2 + (y-2)^2 = .12$
$(x+4)^2 + (y-1.7)^2 = .1 \{y \ge 1.8$


$y = -\frac{8}{5}x + 10.2 \{-3.67 \le y \le -0.9$
$y = \frac{1}{2}(x-2) + 3 \{3.6 \le y \le 7\}$
$y = \frac{3}{2}x + 7 \{3.7 \le y \le 7\}$
$(x+4)^2 + (y-2)^2 = .12$
$(x+4)^2 + (y-1.7)^2 = .1 \{y \ge 1.8$
$-x = -\frac{8}{(y-1.9)} + 2$

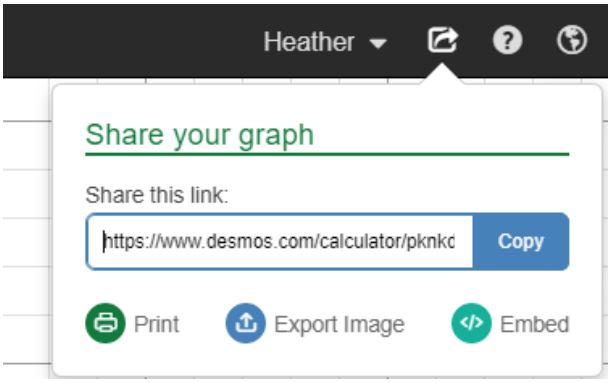




- As students are recreating their picture/art on Desmos, along the way, they are continually reflecting on their mathematical thinking and connecting their picture to functions, relations, and transformations.

- When students have completed a picture they are proud of, they save their picture and send the link to their teacher.

  Link to shark picture: [Desmos Shark Picture]

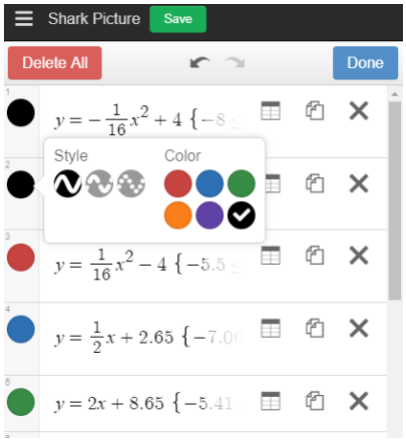


- Students can complete the attached “student assessment” and reflect on their learning experience.
- Now it’s time to celebrate the math! If your school has 3D printers, you can convert these beautiful pictures into STl files, and you can print them.

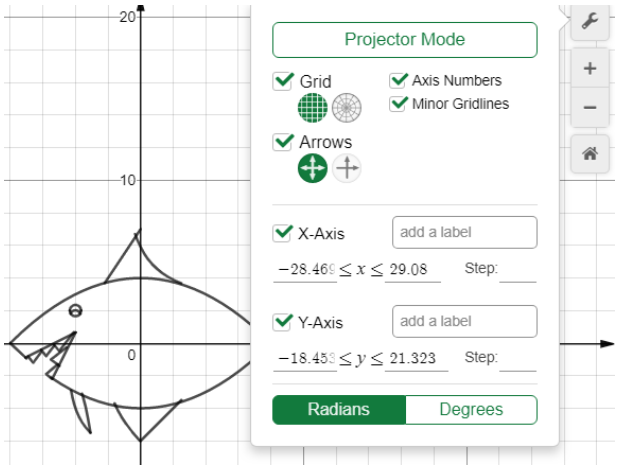
Steps to printing

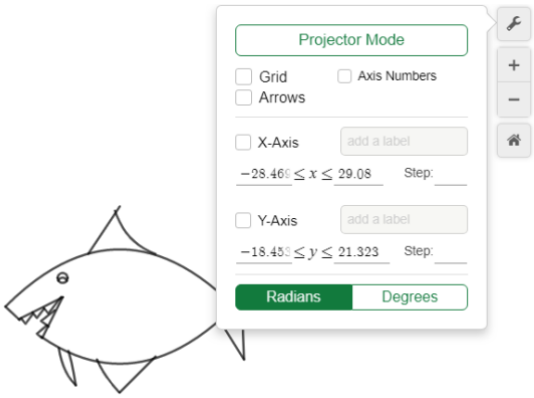
[File Converter]

1. Export a PNG file from Desmos
    - In Desmos, edit your list and make all of the lines black.

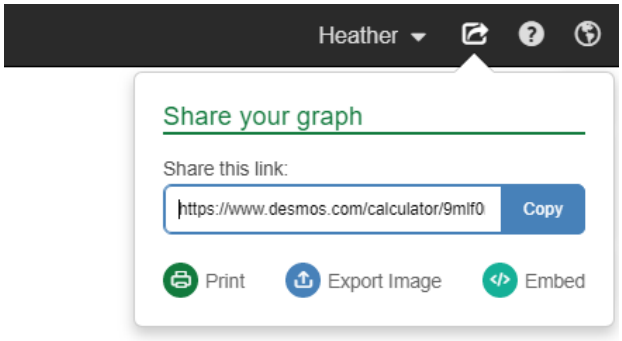


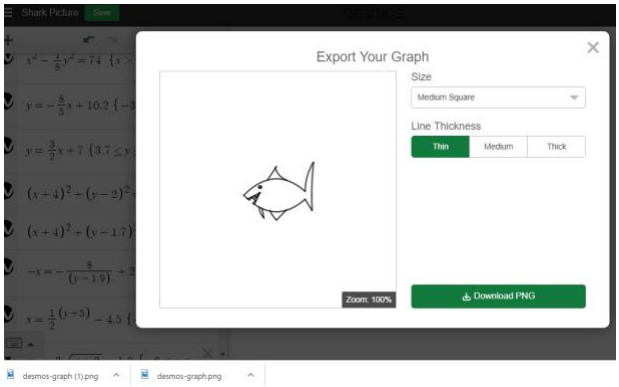
    - Go to graph setting and remove the gridlines from your graph.



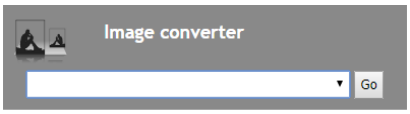


- Export the desmos graph  and download the PNG file.





2. Convert PNG file to an SVG file
   - One way to do this is use [File Converter]

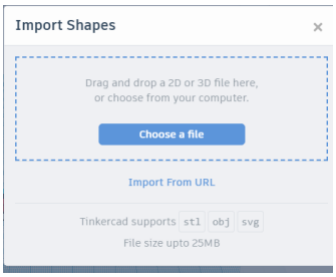
- Go to “image converter” 

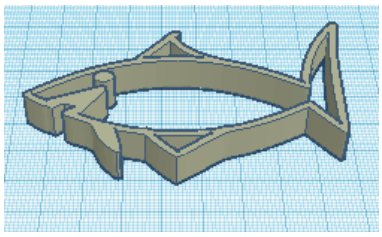
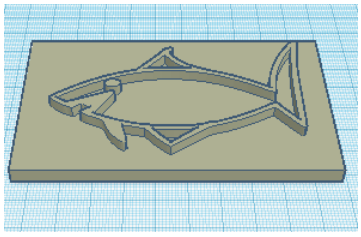
and drop down to “convert to SVG file”

- Upload your PNG file and convert to an SVG file

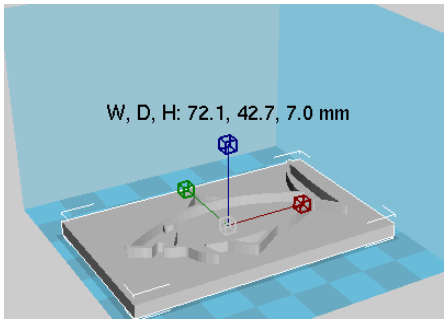
3. Import SVG file into TinkerCAD or a similar program. Add a base if needed and then export as an STL file or similar.

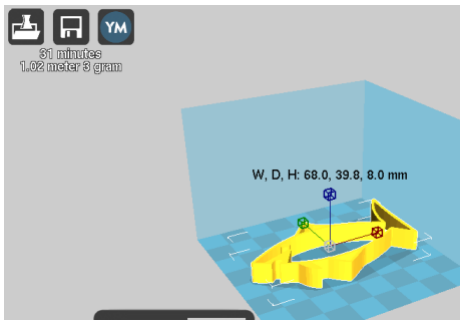
   Video explaining TinkerCad: [Learn TinkerCad]



4. Import into a slicer that goes with the 3D printers that you are using. For example, Cura or Flashprint.





5. Print and celebrate the beauty of math.

Formative Assessment

This is an activity that builds over time, and it's important to always be reflecting on the project. Formative assessment and mathematical conversations between students and the teacher are ongoing to ensure they are connecting functions, relations, and transformations to their picture.

Students are encouraged to share their mathematical thinking, mistakes, and successes with others in the class. Not much additional time is required to facilitate this project in Pre-calculus Math 12 when the teacher can integrate the learning intentions with the math lessons.

Summative Assessment

The "Function/Relation Picture" created from Desmos is overlaid on top of the original image to formally assess this project.

The "Function/Relation Picture" must be scaled appropriately to compare and contrast this recreated picture with the original.

Students provide the equations of functions and relations used to describe this original image.

The demonstration of learning can be achieved with a student presentation or with a teacher-student conference to describe the connections between the original image and their finished project. In this presentation, students will also reflect on their mathematical thinking, connections they made with mathematics and the chosen image, and what they have learned from their mistakes.

Suggested Rubric

4 **I've got this**	**3** **I am understanding this concept**	**2** **I am almost there, but still have more learning to do**	**1** **I have more learning to do**
I have demonstrated a strong understanding of the relations and transformations. I have connected every shape with a relation and I have used all relations. I am aware of my mistakes and what I did to overcome them. I can concisely reflect and communicate about my learning journey. I can explain my mathematical choices.	I have demonstrated an adequate understanding of relations and transformations. I have connected most shapes with a relation and I have used most of the relations. I am aware of my mistakes and what I did to overcome them. I can adequately reflect and communicate about my learning journey. I can explain my mathematical choices.	I have demonstrated a basic understanding of relations and transformations. I have connected some shapes with a relation and I have used most of the relations. I am aware of my mistakes (with coaching) and what I did to overcome them. I can reflect and communicate about their learning journey with coaching. With coaching, I can explain my mathematical choices.	I need more time to work on the following: • Demonstrate an understanding of relations and transformations • Connect shapes with a relation and use more relations • Recognize and overcome my mistakes • Reflect and communicate about my learning journey • Explain my mathematical choices

Suggested Reflection Questions

- What functions or relations did you use?
- What transformations did you use?
- How did making mistakes, if any, contribute to your understanding of functions and relations, transformations, and restrictions of domain and range?
- How did your creativity affect your final project?
- What critical thinking or decisions did you have to make?
- Why did you make these decisions?
- Would you consider doing something different next time?
- Was your original picture too complex to describe mathematically or was it too easy? Explain your answer.
- What connections can you make between art and math?
- How does reflecting on your mathematical thinking deepen your understanding of math?

Extensions

- Bubble Wands — Make them into bubble wands and give them away.
- Mural — Split a picture and get the whole class to do a piece (differentiate learning). This is also an effective way to limit plagiarism.
- Encourage the use of a variety of cultural art styles.
- Indigenous Art — Speak to your Indigenous education support team and use local Indigenous art in the introduction. You must get permission from the artist to use their image.
- 3D Printer — Make stamps out of the 3D printed objects and do stamp art or other 3D objects
- Connect with a Kindergarten or Grade 1 class. Students could make Desmos art from the younger kids' work and deliver the finished product, 2D or 3D.

Note: There are a multitude of Desmos drawing project examples online, and some students are tempted to represent one of these as their own. One way to combat this problem is for the students to "trace" a simple drawing using domain restricted functions — that is approved by the teacher beforehand. This does not completely eradicate plagiarism. Including a schedule of the number of and type of functions that can be used — especially severely limiting the number of linear equations — effectively limits plagiarism.

Writer's Comments

This is my absolute favorite project for Pre-calculus 12. It allows students and teachers to see and celebrate the beauty of math. Students end up with a final product that they can show others. In our school, we celebrated math by having a bulletin of this project and a display in the library. This was our first ever math display in the library.

If there is a 3D-printer in your building, I would highly recommend 3D printing the end products. 3D printers may seem scary but they are fairly simple to operate. I am lucky enough to have two 3D printers in my classroom. I do all the maintenance on my 3D printers. I was able to learn everything by talking to colleagues and watching YouTube videos. I found that students loved seeing their projects getting printed. For most of my Pre-calculus 12 students, this was their first exposure to 3D printing.

The Core Competencies to name, notice and nurture are creative thinking and critical and reflective thinking. Students use creative thinking skills in picking out or drawing the picture they are going to create. They use both creative and critical and reflective thinking to figure out what relations will work for their picture. I talk about the Core Competencies all the time in my classes. The First Peoples Principles of Learning covered are; "Learning involves patience and time." and "Learning is holistic, reflexive, reflective, experiential, and relational". This project takes patience and time. During the project, students have to constantly reflect and experiment to figure out how to manipulate the functions to create their picture.

When I started doing this project, I did not give any introduction. I just asked students to draw a picture in Graphing Desmos using the functions we were learning. I noticed that many students seemed to struggle at the beginning of this project. After talking with students, I discovered they often seemed to produce better products and struggle less at the beginning if they had a picture in mind.

I have always introduced this project at the beginning of the course. Any time students have completed their work, they can work on the project. Next time I do this project I am planning to do as a review near the end of the course, so trig functions can be added. I think adding trig functions would be really fun. They are so good for hair, noodles and ocean waves.

I have done this project with Grade 11's as well. They are required to use parabolas and lines to create a picture.

Thanks for reading this lesson. I hope you have fun playing with relations and making beautiful math.