

Apprenticeship and Workplace Mathematics 10

Introduction Assignment

Student Name _____

Student No. _____ Date _____

Address _____ Postal Code _____

Based on the instructions provided by your school, complete the following *Apprenticeship and Workplace Mathematics 10* assignment and return it to your teacher.

Contents:

40 pages

Assignment time:

2 hours

Lesson

Spatial Reasoning Puzzles and Games

To complete this lesson, you will need:

- scissors
- an old file folder or cardstock on which to glue the templates
- a glue stick

In this lesson, you will complete:

- 5 activities
- a two-part Assignment



Essential Questions

On the next page, you'll find the Essential Questions for this lesson. Fill out the first column before you start working on the lesson. This will help you think about what you already know about the lesson topics.

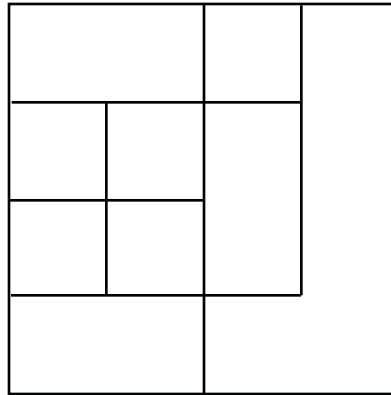
When you're finished the lesson, you will be prompted to go back to the Essential Questions page and fill in the second and third columns.

You will submit the Essential Questions with your Introduction Assignment.

| ESSENTIAL QUESTIONS | Before the Lesson: What I Know | After the Lesson: What I Learned | Examples |
|--|-----------------------------------|-------------------------------------|----------|
| What does “spatial reasoning” mean? | | | |
| What are some strategies you can use to solve puzzles and win games? | | | |

Focus

How many squares can you find in the pattern below?



We'll revisit this puzzle and reveal the solution in the Lesson Summary. For now, think about the strategies you would use to solve the puzzle.

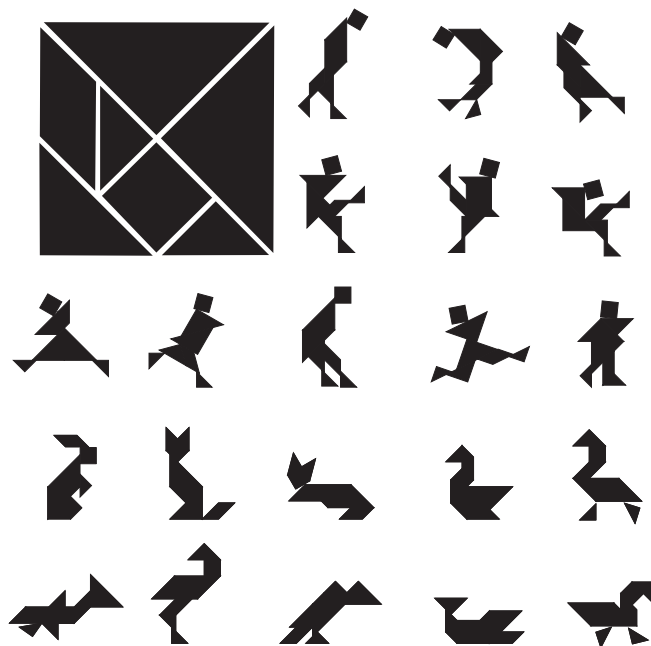
Maybe you would shade in or outline the squares as you find them. You might redraw the puzzle several times so your shading or outlines don't get mixed up.

If you prefer a methodical approach, you might try to find all the squares with an area of 1 cubic unit first. Then you could look for all the squares with an area of 4 cubic units, then 9 cubic units, and finally 16 cubic units.

Can you think of another strategy? Record it here:

In this lesson we will explore puzzles and games that involve visual designs and geometric shapes. As you work through this lesson, think about the strategies you use to solve puzzles or win games.

Get Started



The tangram is a popular puzzle thought to have originated in China well over two centuries ago. This type of puzzle was introduced to the Americas at the beginning of the 19th century. The tangram is still popular here in Canada and elsewhere.

Part of the tangram's popularity comes from its union of geometry and art. The puzzle pieces, which fit together to form a square, can be rearranged to form countless shapes and figures, as illustrated in the image above. What figures can you make using these shapes? You'll have a chance to try it out in the next activity!

Activity 1 Try This

In this activity you will investigate the tangram puzzle. You may work with a partner.

First, you must create the playing pieces. Follow the instructions below.

Step 1: Get the “Tangram Template” from the Appendix at the end of this assignment.

Step 2: Glue the Tangram Template to the back of an old file folder or to a piece of heavy cardstock.

Step 3: Carefully cut out the seven pieces. You will end up with five triangles, one square, and one parallelogram.

Now that you have cut the pieces apart, see if you can form the tangram shapes shown.

Rules of the game:

- You must use all seven pieces to form each shape.
- You cannot overlap the pieces, but you can rotate pieces and even flip them over.

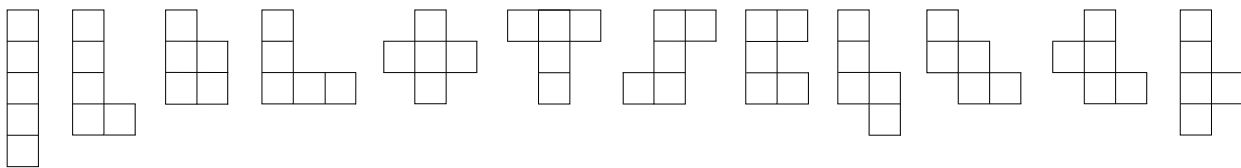
See how many of the shapes you can create!

Explore

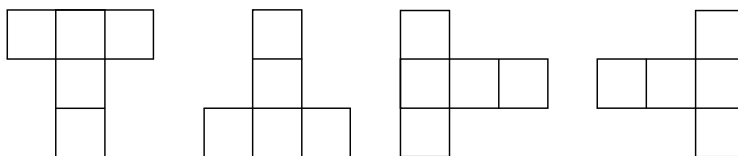
You have just investigated the tangram puzzle. That puzzle did not require you to use numbers or equations. Instead, you had to visualize patterns and manipulate shapes in order to solve the puzzle. The type of reasoning involved when you solve a problem this way is called **spatial reasoning**.

Pentominoes

Another popular game and puzzle which involves spatial reasoning is pentominoes. The word *pentomino* describes a shape formed by joining five identical squares. Each square in the set of five shares at least one side with another square. There are 12 such arrangements. These arrangements make up the pieces of the pentomino game.



It may seem like there should be more arrangements. However, because the pentominoes can be flipped over and rotated during play, the number of unique pieces is limited. The following arrangements, for example, can all be made with the same pentomino.



Activity 2

Try This

In this activity you will play a game of Pentominoes. You'll need a partner or two. Ask your friends or family members to play! You can play this game with 2, 3, or 4 players.

First, you'll need to construct the pieces and the board.

Step 1: Get the "Pentomino Pieces Template" from the Appendix at the end of this assignment.

Step 2: Glue the "Pentomino Pieces Template" to the back of an old file folder or to a piece of heavy cardstock.

Step 3: Carefully cut out the twelve pieces. Since the game pieces can be flipped over, you may wish to draw the black grid lines on the back of each piece as well.

Step 4: Get the "Pentominoes Game Board" from the Appendix. You may want to glue the board to a piece of cardboard to keep it from moving during play.

Now that you have the pentominoes and the game board, you're ready to play! Here are the rules:

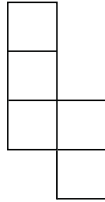
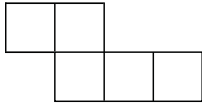
This is a game for 2, 3, or 4 players.

- Players take turns placing a pentomino of their choice anywhere on the game board. The pieces may be rotated or flipped over, but they must be placed on unoccupied squares on the board.
- Pieces may touch, or even be placed side by side, but they must not overlap!
- If there are two players, the first player who cannot place a piece on the board, loses.
- If three or more players begin, the first player who cannot place a piece on the board is eliminated. The board is cleared, and the remaining players play a second round. Players are eliminated until a winner is declared!

Play several games until you are confident that you can describe your game strategy.

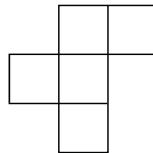
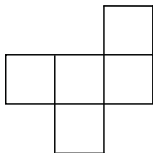
Activity 3 Self-Check

1.



Do the two figures above represent the same pentomino piece? Why or why not?

2.



Do the two figures above represent the same pentomino piece? Why or why not?

3. If you were coaching a younger brother or sister to play the pentomino game, what strategies would you recommend they use to win the game?

4. a. In Activity 2 you worked with at least one partner. Ask that partner what their strategy was and record it here.

- b. Compare your partner's strategy with yours. Which strategy do you think would be more effective?



Turn to the solutions at the end of the lesson and check your work.

Bringing Ideas Together

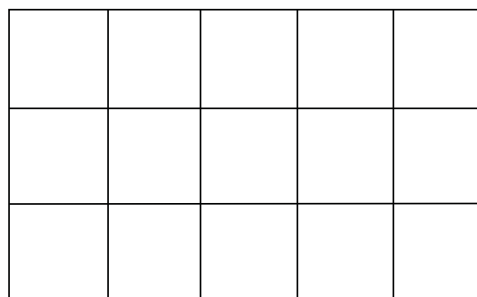


In Explore you investigated strategies for playing pentominoes on a game board consisting of an 8×8 grid. There are 64 squares on the game board. Because each of the 12 pentominoes is made up of 5 squares, it is possible to place all 12 on the board with 4 squares left uncovered. $12 \times 5 = 60$ $64 - 60 = 4$

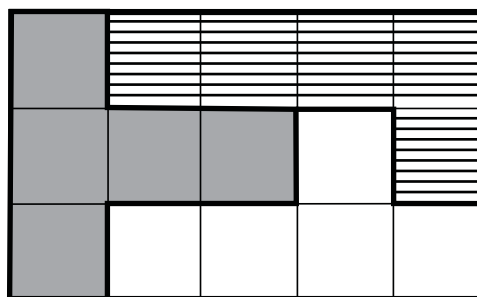
There are many variations on the game of pentominoes that you played in Activity 2. For example, some are puzzles in which you are asked to completely cover a rectangle or square with pieces chosen from your set of 12 pentominoes. You can rotate the pieces you select, and you can flip them over, but you can't overlap them.

Example 1

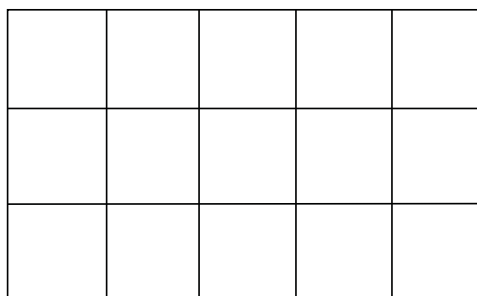
- How many pentominoes are needed to completely cover a rectangle with an area of 15 squares?
- Find at least one solution to this puzzle.

**Solution**

- There are 15 squares to cover. You will need 3 pentominoes, since $3 \times 5 = 15$.
- Here is one solution.



There are many other solutions to this puzzle. Use your pentominoes to find at least one more. Record your solution below.



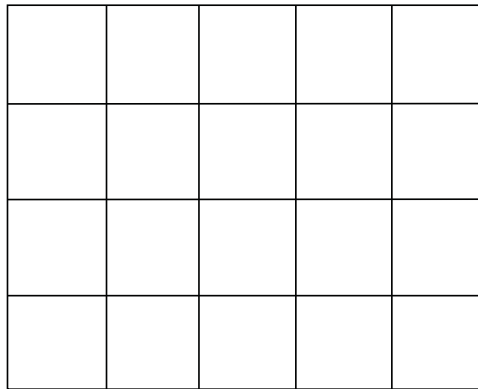
If you have access to the Internet, you can find interactive pentomino puzzles online. Start your search at the AWM 10 Companion Website!
<http://www.openschool.bc.ca/courses/math/awm10>

Activity 4

Self-Check

Please complete the following questions.

1. Find at least one way to cover a 4×5 grid completely.



2. List three sizes of grids which you might be able to completely cover.

3. What would the area of the largest grid be that could be completely covered with your 12 pentonimoes?



Turn to the solutions at the end of the lesson and check your work.

Activity 5

Mastering Concepts

When you have a problem to solve, what strategies do you use? Here are some common problem-solving strategies:

- identify knowns and unknowns
- trial and error
- look ahead and anticipate
- work backwards
- apply similar processes
- look for patterns
- systematic list
- draw or model
- eliminate possibilities
- simplify the problem—get rid of unneeded data
- search the Internet for help

1. Which strategies did you use during this lesson?

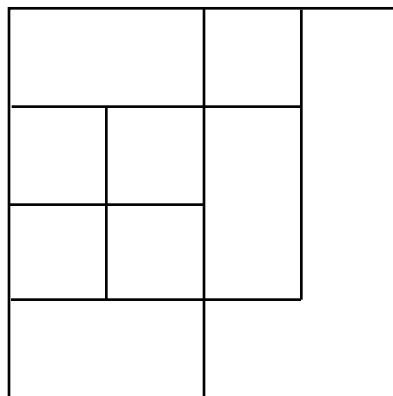
2. Describe another situation where you've used one of the listed strategies to solve a problem. (Your situation doesn't have to be a math problem!)



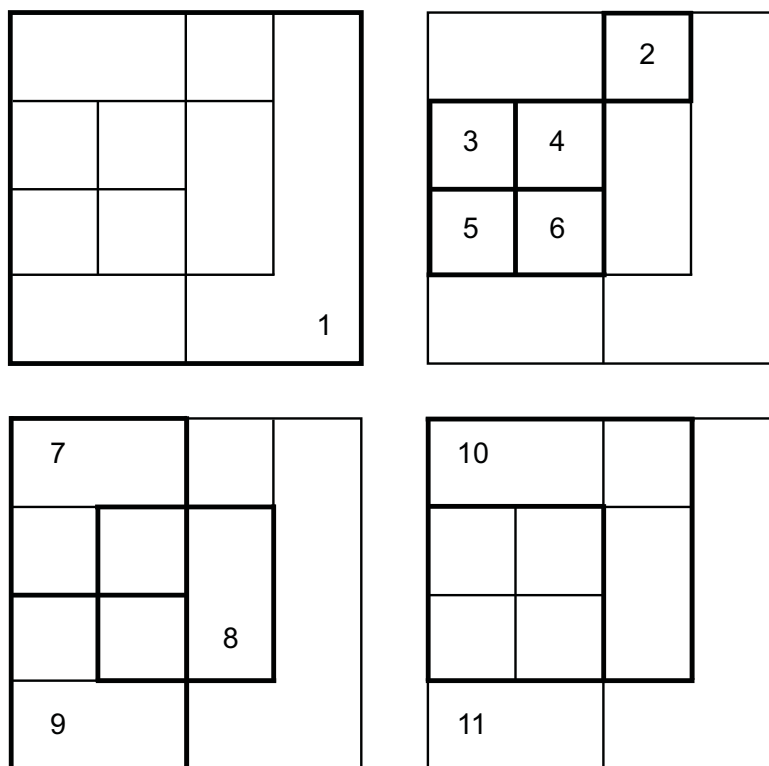
Turn to the solutions at the end of the lesson and check your work.

Lesson Summary

At the beginning of this lesson you were asked: how many squares can you find in the pattern below?



Were you able to find all 11 squares? Check out the following solution.



In this lesson you explored some puzzles and games that involve visualizing patterns and manipulating shapes. These puzzles and games are designed to challenge your spatial reasoning skills. There are lots of strategies that you can use to solve these puzzles; the trick is finding the ones that work best for you!



There are many examples of spatial puzzles and games on the Internet. If you enjoyed the activities in this lesson, you might want to look for more examples online. You can start your search at the AWM 10 Companion Website (<http://www.openschool.bc.ca/courses/math/awm10>)!



Now it's time to return to the Essential Questions from the beginning of the lesson and complete the final two columns. You'll be handing it in with your Introduction Assignment.



Please complete **Parts 1 and 2** of your **Introduction Assignment** now.

MARKS

**Introduction Assignment:
Part 1**

1. Spatial reasoning is an important skill in many professions. List one hobby or career in which spatial reasoning would be useful. Justify your choice with a brief explanation. (2 marks)

2. Sketch a figure of your own design using all seven tangram pieces. It must be different from the figures shown in the Focus of this lesson. Label your design with an appropriate title that describes the pattern or object that you have created. (3 marks)

Tip: Lay the back of this page on your desk and use your tangram pieces to create your shape on top of the page. Then, trace around each of the pieces as they are arranged on the page.

INTRODUCTION ASSIGNMENT

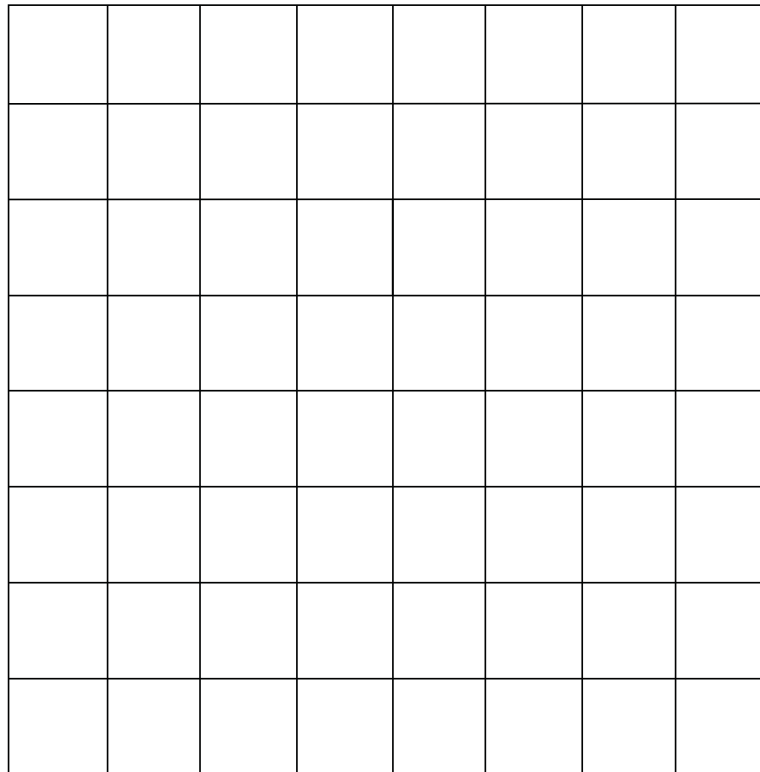
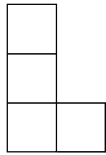
MARKS

MARKS

Use the following information to answer Questions 3 and 4.

Use what you know about pentominoes to try a new puzzle! Instead of playing pentominoes, you will design *tetrominoes*. The prefix *tetra* means *four*. Tetrominoes are shapes formed by joining four squares so that they touch on at least one side.

3. There are only five unique tetrominoes since the figures can be rotated or flipped over during play. One tetromino is shown below. Sketch the other four possible tetrominoes. (2 marks)



MARKS

4. Suppose you cut out two of each of these five shapes—you would then have ten pieces in total. Design a game board on which you would cover a rectangular grid with some, or all, of the tetromino pieces. Two pieces of grid-paper are provided in the Appendix for you to use.

- a. What are the dimensions of your gameboard? Explain why you chose this size. (2 marks)

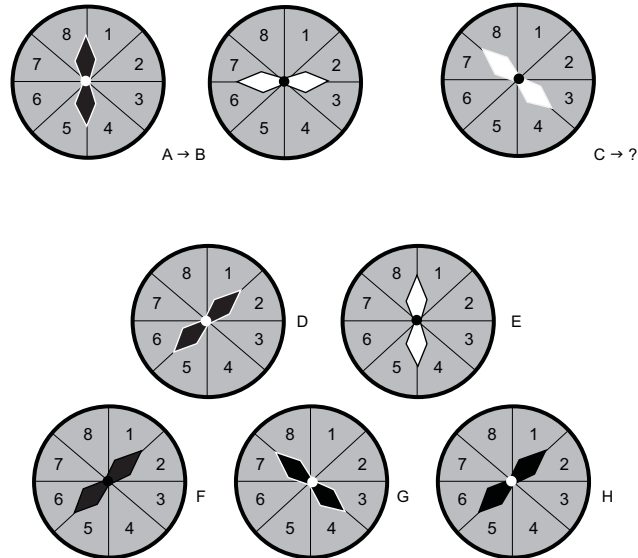
- b. Include one possible solution to the puzzle using your gameboard. (2 marks)

MARKS

Use the following information to answer Question 5.

Have a look at the puzzle below, and the solution that a student provided.

Puzzle



Look at the relationship between spinner A and B. Identify which spinner (D–H) should be paired with spinner C so that C and its partner have the same relationship as A and B.

Solution

To go from A to B, you must:

- rotate the pointer 90°
- change the colour of the pointer
- change the colour of the dot at the centre of the pointer

If you perform these actions on spinner C, you will find that D is the partner for C.

5. The student's solution is incorrect. Identify the student's error, and describe the correct solution. (2 marks)

INTRODUCTION ASSIGNMENT

MARKS

| | |
|------------|--------------|
| <hr/> | Total |
| /14 | |

MARKS



Introduction Assignment: Part 2

For this part of the assignment, please choose EITHER Option A or Option B. Do **not** complete both.

Option A

In this assignment, you'll be asked to find an existing puzzle or game, describe it, and describe the strategy you used to solve the puzzle or win the game.

You may search on the Internet or in books to find a puzzle or game that you like. The puzzle or game must require you to use spatial reasoning to solve or win it.



There are lots of puzzles and games online. Start your search at the AWM 10 Companion Website (<http://www.openschool.bc.ca/courses/math/awm10>)!

Once you've found a puzzle or game, and at least *tried* to solve or win it, please answer the following questions.

1. a. What is the puzzle/game called and where did you find it? (Please cite the title/author of the book, or the URL of the website.) (1 mark)

-
- b. Briefly describe the puzzle/game. (1 mark)

MARKS

2. Why did you select this puzzle/game? (2 marks)

3. Did you solve the puzzle correctly or win the game? (It's okay if you didn't, as long as you tried!)

circle one: yes no

If you answered **yes**, describe your solution and the strategy you used to reach the solution.

If you answered **no**, describe the strategy you used and where you got stuck. (3 marks)

MARKS

Option B

In this assignment you'll be asked to create your own puzzle or game, describe it, and provide a solution to the puzzle or the game.

Do you have a great idea for a puzzle or game that will stump your friends? Here's your chance to create it! The puzzle or game must require the player to use spatial reasoning to solve or win it.

Once you've created your puzzle/game, answer the following questions.

1. What is the name of your puzzle/game? (1 mark)

2. If you created a puzzle, please present it or describe it here, and include the solution. If you created a game, describe here the object of the game and the rules of play. (4 marks)

INTRODUCTION ASSIGNMENT

MARKS

3. Describe a strategy someone could use to solve your puzzle or win your game. (2 marks)

/7

Total

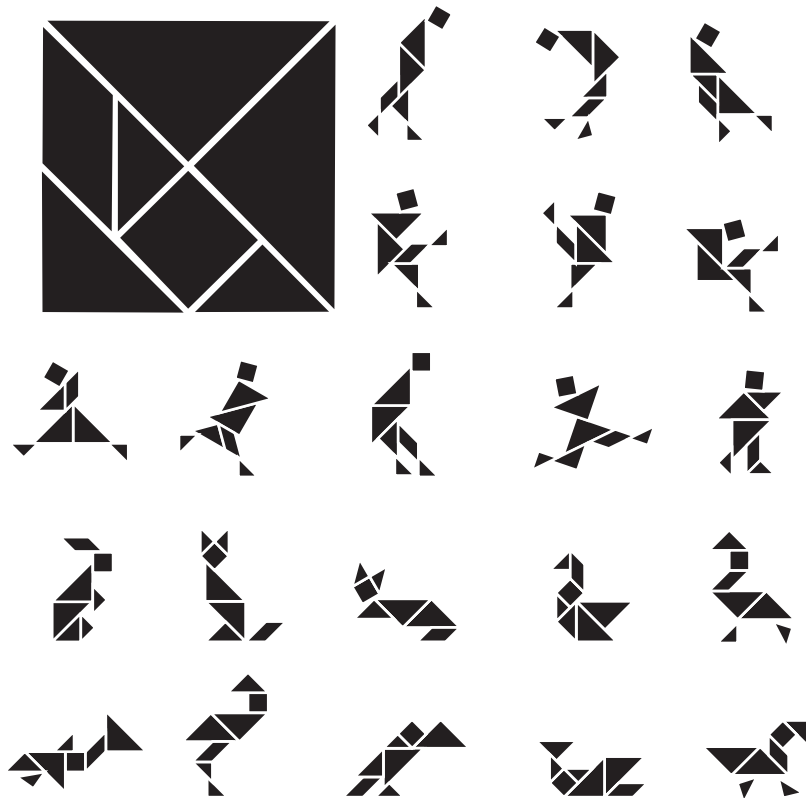


You have now finished the Introduction Assignment. When you submit it to your teacher, make sure you have also included all of the assignment pieces.

APPENDIX

Introduction Assignment Activity Solutions

Activity 1: Try This



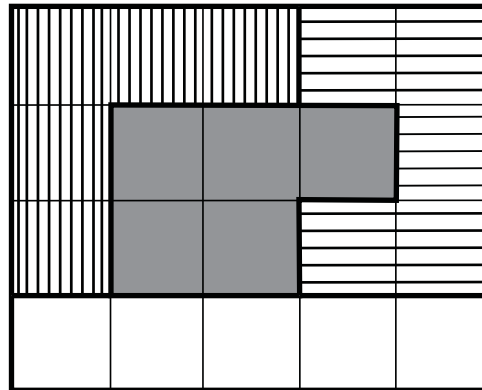
Activity 3: Self-Check

1. The pieces are the same. To obtain the second piece from the first, rotate the first piece 90° counterclockwise, then flip the piece over.
2. The drawings of the pieces represent the same pentomino piece. To obtain the second drawing from the first, rotate the first drawing 90° counterclockwise, then flip the rotated drawing over.
3. Be generous. Play your pieces so that they adjoin pieces played by your opponent until the end of the game approaches. Then look at what pieces remain and select the piece you think can be positioned in the remaining vacant squares so that it will be impossible for your opponent to place a remaining piece on the board.

4. a. Answers will vary.
- b. Answers will vary.

Activity 4: Self-Check

1. Answers will vary. A sample answer is given.

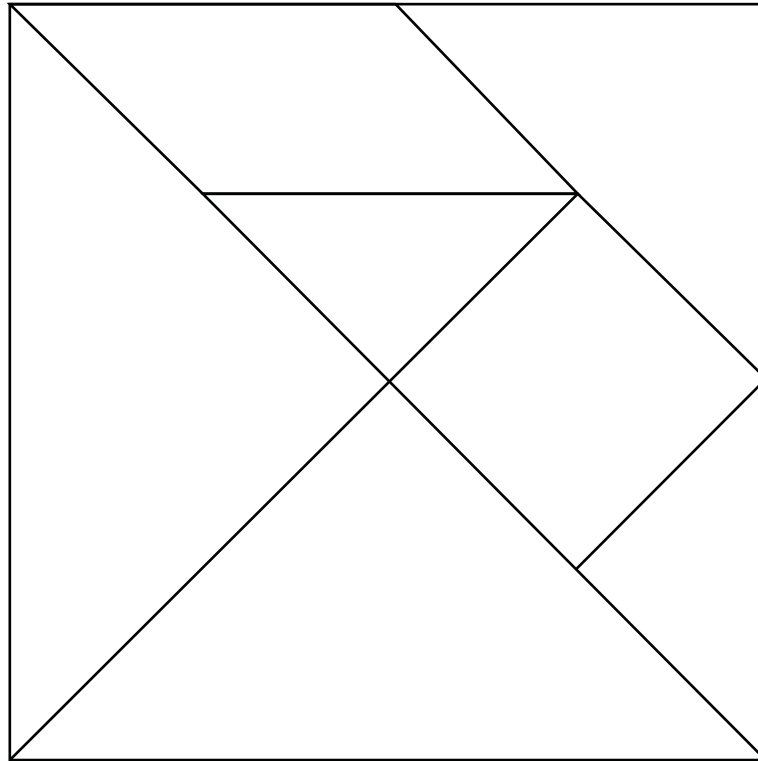


2. You have already covered 15 square and 20 square grids. Other grids you might try are grids with areas of 25, 30, 35, 40, 45, 50, 55, and 60 squares.
3. Since $12 \text{ pieces} \times 5 \text{ squares} = 60 \text{ squares}$, a grid of 60 squares is the largest you could cover. Some 60 square grids are 5-by-12, 6-by-10, and 3-by-20.

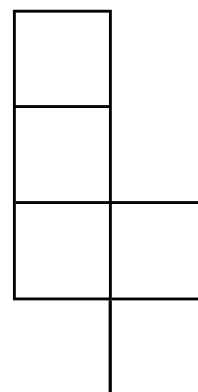
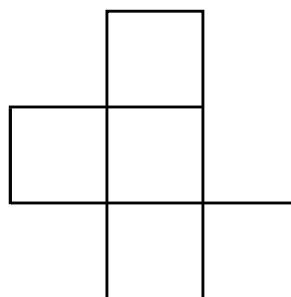
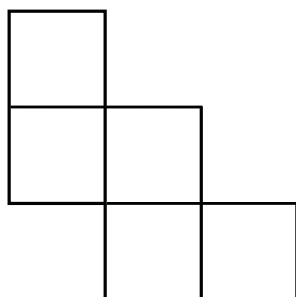
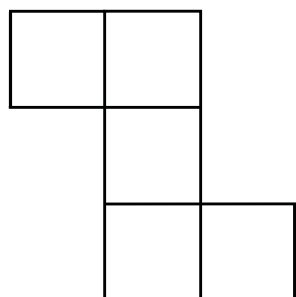
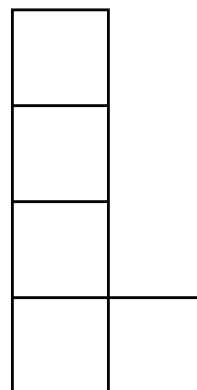
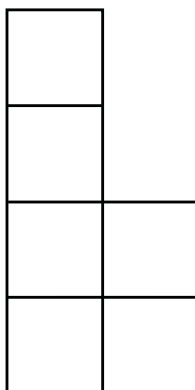
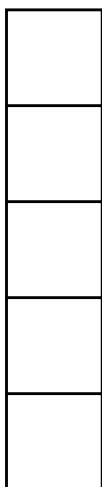
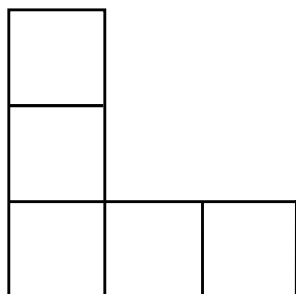
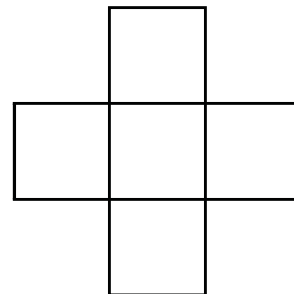
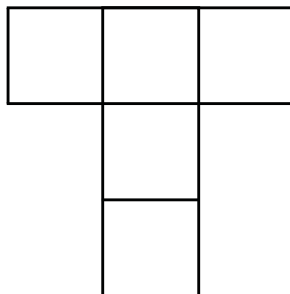
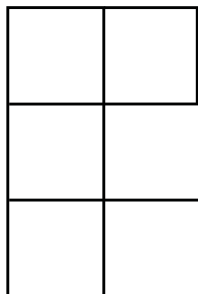
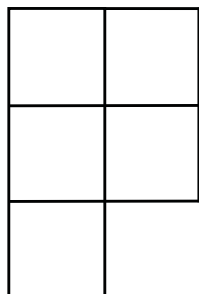
Activity 5: Mastering Concepts

1. You may have looked for patterns as you played with the tangrams. You likely used trial and error as you tried to find a winning strategy to the pentaminoes game. You probably even looked ahead and anticipated moves during this game. You probably used many of the strategies listed without even realizing!
2. Answers will vary.

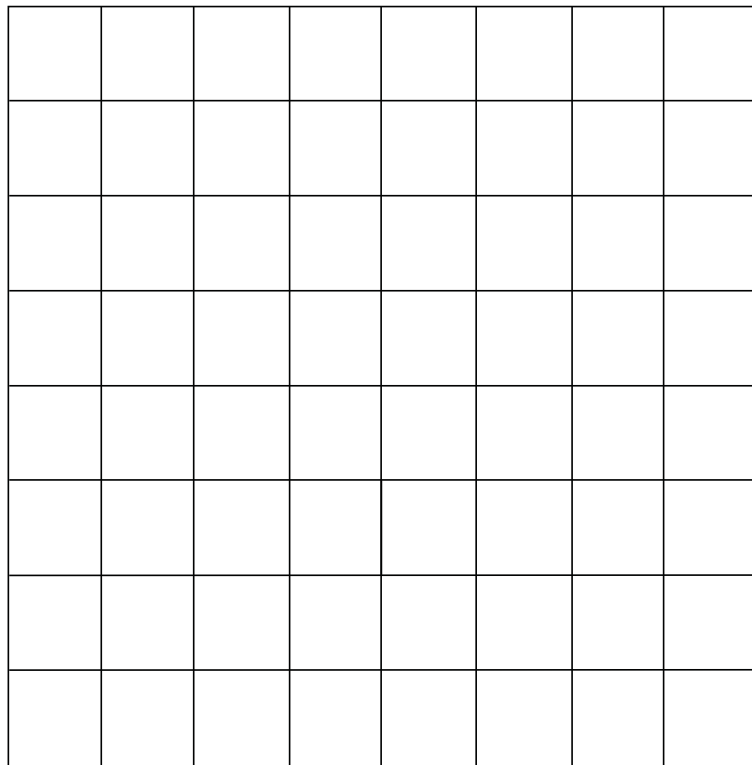
Tangram Template



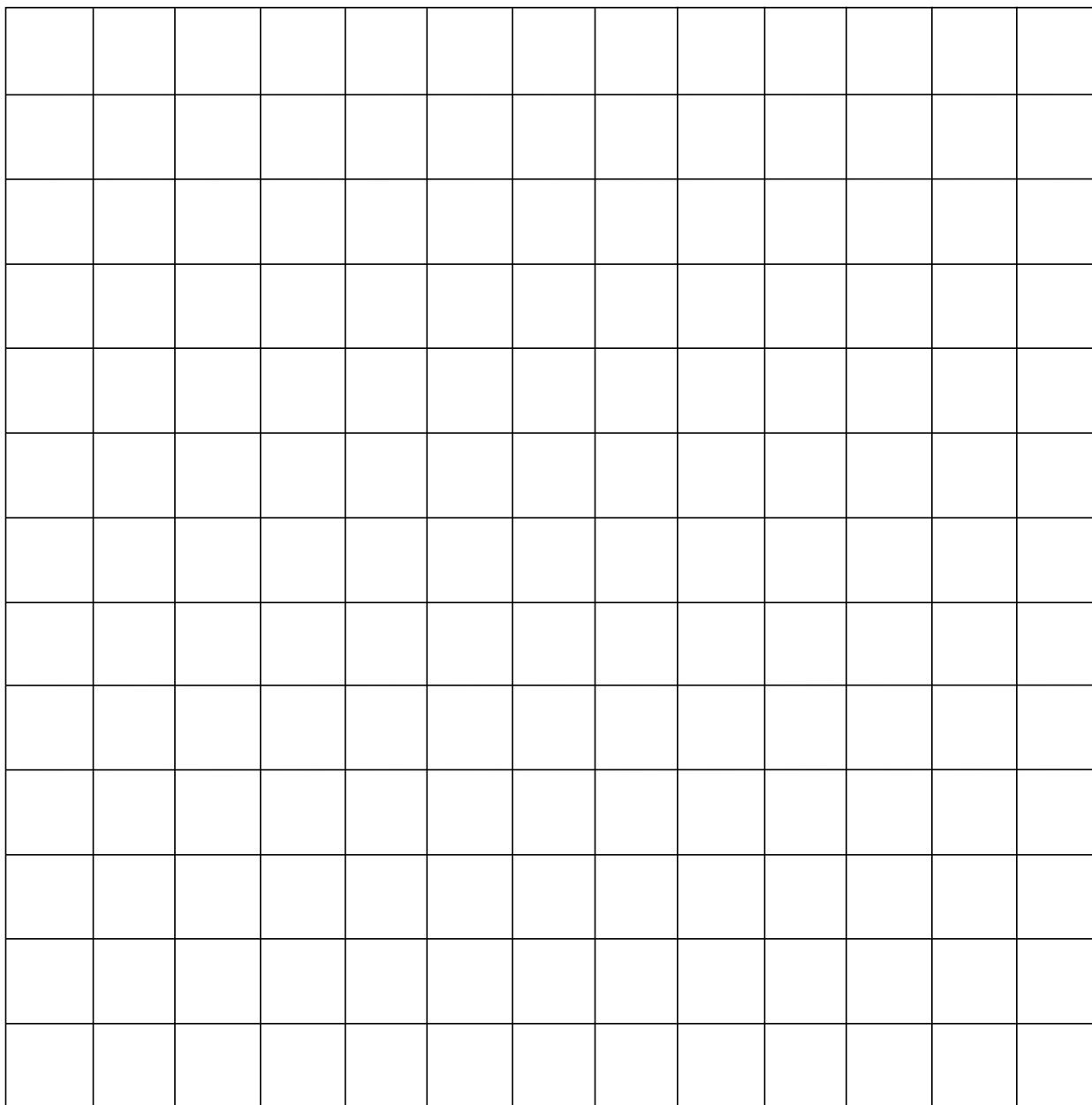
Pentomino Pieces Template



Pentomino Game Board



Assignment Part 1 Grid



Assignment Part 1 Grid

